Note 6.2 - Trigonometric Integrations

1 Introduction

In this note, we first deal with integrals involving powers of trigonometric functions. We have already known two of them:

With a little trig identities, we also know

These will be the basic integrals we will use to derive more general formula. We will apply these skills, with the square sum identities, to integrate functions involving some unfriendly terms - the square roots.

2 Trigonometric Identities

Let's review some basic trigonometric identities that will used

3 Instruction Manuals - Product of Trig. Functions

3.1 $\sin^n x \cos^m x$

The best case is in fact when one of n or m is odd:

When both n and m are even, $\sin^n x \cos^m x$ can be turned into a polynomial of $\cos^2 x$:

and each term can reduced further and we eventually end up with

We then integrate the above function term-by-term.

3.2 $\sec^n x \tan^m x$

The substitution is most likely $u = \sec x$ or $u = \tan x$. It is probably easier to just pick one substitution and see if it works:

4   Examples

5   Instruction Manuals - $\sqrt{x^2 - a^2}$, $\sqrt{a^2 - x^2}$, $\sqrt{x^2 + a^2}$

The universal first step for functions involving these terms is obvious - *get rid of square roots!* To do it, we make whatever inside $\sqrt{}$ into a square:

After the removal (and proper substitutions), we end up with a rational functions of trigonometric functions that we integrate, with possible help from section 3.

6   Example