Note 7.1 - Area and Volume

1 Introduction

We apply integration to derive (or verify some familiar) area/volume formula for certain two/three dimensional objects. Throughout the applications, we stick to the principles that integration is the sum of infinitely many familiar and simple objects. In another words, we want to decompose the big objects in a way that each small piece is a quantity we can easily compute and describe them by functions. Then we sum them up by integrating those functions.

2 Area Between Curves

By now we all know that integral is the signed area under (above) the curve. Consequentially, we can compute the area *between* graphs of functions by taking appropriate differences:

3 Volume by Cross Sections

For a three dimensional object, we usually compute the volume by cutting it into *thin plates*, each of thickness dx or dy and therefore of volume $A(x)dx$ or $A(y)dy$:

The volume is then

$$V = \int_a^b A(x)\, dx, \qquad \text{or } V = \int_c^d A(y)\, dy$$

where the limits of integrations are determined by given information.

4 Examples