Note 3.2 - Differentiation Properties and Formula

1 Introduction

We now develop some formula for the derivatives of elementary functions. The nice thing about differentiation is that every computation has formula and rules to follow. Of course, it comes with the price that differentiation is hard to define. We will soon see that integration is the opposite (easy to define but hard to compute).

2 Properties

As expected, differentiation is a linear operator:

It is not hard to prove it from definition and the linearity of lim.

However, differentiation does not commute with multiplication nor division:

Instead, we have separate rules to treat derivatives of product and quotients.

3 Product Rules

Theorem 3.1. *Let $f(x)$ and $g(x)$ be differentiable functions, then $h(x) = f(x)g(x)$ is differentiable and*

$$h'(x) = f'(x)g(x) + f(x)g'(x).$$

We explain this rule by the following picture:

With this, we can derive the derivative for $f(x) = x^n$, with $n \in \mathbb{N}$.

Therefore, the derivative of a polynomial is:

4 Quotient Rules

To derive the derivative for $h(x) = \frac{f(x)}{g(x)}$, we first derive the derive for

$$g(x) = \frac{1}{f(x)}$$

for differentiable f with $f'(x) \neq 0$.

Then we use the product rule to derive $h'(x)$ above:

5 Trigonometric Functions

We only need to derive the derivative for $\sin x$ and $\cos x$, since all the other trigonometric functions are algebraic combinations of these two basic functions.

$$\frac{d}{dx}\sin x =$$

$$\frac{d}{dx}\cos x =$$

Let's try to derive differentiation formula for other trigonometric functions:

6 Examples