Note 4.3 - Curve Sketching

1 Introduction

We introduce another important application of differentiation - the sketching of the curve $y = f(x)$. We have done this precisely for lines $f(x) = ax + b$, and somewhat precisely for low order polynomials. However, as the functions get more abstract, we do not have explicit ways to determine basic information like monotonicity (curves going up or going down), turning points, or shapes ... etc. However, by now we have understood that they are determined by derivatives. Let's lay them out explicitly.

2 The General Steps

Let's take the more familiar example of sketching $y = f(x) = x^4 = 4x^3 + 4x^2$ for $-1 \leq x < 5$. Here are the general steps:

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3 The Concavity

There are two distinct ways for a curve to rise or fall:

For a continuous curve, this can be describes as turning left or right along the curves. If the function is differentiable so that we have tangent lines along it, this is precisely described by the monotonicity of the first derivatives:

If, moreover, the function is twice differentiable, the monotonicity of f' is the sign of f''. Therefore, we conclude

4 Asymptotes

We sometimes have to deal with *unbounded domain* or *unbounded function*. For the first situation, we need to check whether $\lim_{x\to\pm\infty} f(x)$ exists. If it does, $y = f(x)$ has a *horizontal asymptote*:

If the limit does not exist, then we need to know which side the function blows up:

For the situation where $f(x)$ blows up at a certain point c, we have a *vertical asymptote* at $x = c$. We need to determine how the function blows up from either side of the vertical line:

5 Examples

Let's practice with some examples.