Note 5.3 - Properties, Formulas, and Computations of Integrations

1 Introduction

We will list some basic formula and properties of integration. We will see that integrations, even though started from elementary questions and definitions, are not easy to compute. In fact, we can only find antiderivatives for a very limited class of functions.

2 Basic Properties

The only algebraic rule available for integration is the linearity:

A very important property of integration is the *monotonicity*:

Indeed, larger (smaller) function has larger (smaller) area underneath it. Here are some consequences of monotonicity:

Let's discuss the sign of integrations and some consequences.

3 Formula

Let's reverse the differentiation formula of elementary functions and list some basic antiderivatives:

Some basic examples:

4 Change of Variables

There are very few universal rules for integration: functions have to be special for us to guess their antiderivatives. One example is when the integrand contains a function u and its derivative u'. The term dx represents the infinitesimal change in x and therefore the "denominator" of $\frac{df}{dx}$. By chain rule, we have

So it may help to rewrite $f(x)dx$ into $g(u)du$ (as well as the limits of integration) and hopefully $g(u)$ is some function we can integrate more easily. Let's look at some examples: