High-Order SBP Implicit Difference Operator

§ 1D Test System Problem

Consider the advection system:

$$\begin{cases} q_t + Aq_x = 0, x \in I = [0,1], t \geq 0 \\ u(x,0) = v(x,0) = sin(2\pi x) \\ q(0,t) = (sin(-2\pi t)\ ,\ sin(-2\pi t))^T \\ q(1,t) = (sin(2\pi(1-t))\ ,\ sin(2\pi(1-t)))^T \end{cases} ,\ q = \begin{pmatrix} u(x,t) \\ v(x,t) \end{pmatrix},\ A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

, which has the exact soltion

$$q = \begin{pmatrix} sin(2\pi(x-t)) \\ sin(2\pi(x-t)) \end{pmatrix}$$

The below convergence results of this test problem, is measured at terminal time $T = 1$, $CFL = 0.85$ and $\tau = 1$ for 77451-scheme, $CFL = 0.8$ and $\tau = 1.1$ for 99681-scheme.

	77451		99681	
N	$\parallel \delta V(N) \parallel_{l^2}$	α	$\parallel \delta V(N) \parallel_{l^2}$	α
21	$2.957132e-004$	-	$2.396580e-005$	-
41	$6.962447e-006$	5.6033	$1.661233e-007$	7.43
81	$2.009329e-007$	5.2070	$1.173269e-009$	7.27
121	$2.473702e-008$	5.2192	$6.599747e-011$	7.17
161	$5.577425e-009$	5.2153	$8.634793e-012$	7.12

The errors in $l^2 - norm$, $\parallel \delta V(N) \parallel_{l^2} = \sqrt{\sum_{i=0}^{N} |V_i(t) - q(x_i,t)|^2 \Delta x}$, and the order for convergence is defined by $\alpha = \log\left(\frac{\|\delta V(N_1)\|_{l^2}}{\|\delta V(N_2)\|_{l^2}}\right) / \log\left(\frac{N_2}{N_1}\right)$. The two figure below, are the results of long time ($T = 1000$) process, with $CFL = 0.85$, $\tau = 1.1$ and $\tau = 1.2$ for 77451-scheme and 99681-scheme respectively.