Elementary Numerical Partial Differential Equations
Homework 2

(Due: Jan. 24, 2008)

Consider the parabolic partial differential equation

$$
\begin{aligned}
u_t &= \epsilon u_{xx}, \quad 0 < x < 1, \quad 0 < t < .5, \\
u(0,t) &= u(1,t) = 0, \quad 0 < t < .5, \\
u(x,0) &= v(x), \quad 0 < x < 1,
\end{aligned}
$$

where $\epsilon > 0$. Recall that if $v(x) = \sin \pi l x$, then the exact solution is

$$
u(x,t) = e^{-\pi^2 l^2 \epsilon t} \sin \pi l x.
$$

Consider uniform refinement, that is, for $h = 1/N$ and $k = 1/M$, we let $x_j = jh$ and $t_n = nk$. Write a Matlab program to solve the equation with the finite difference schemes (12.5) and (12.6).

- Study the performance (stability and accuracy) of the schemes, compare the computed results with the exact solution and discuss the difference.