Numerical Partial Differential Equations I
Homework 3

(Due: Dec. 27, 2006)

Consider the parabolic partial differential equation

$$\begin{aligned} u_t = au_{xx}, &\quad 0 < x < 1, \quad 0 < t < 1, \\ u(0,t) = u(1,t) = 0, &\quad 0 < t < 1, \\ u(x,0) = v(x), &\quad 0 < x < 1, \end{aligned}$$

where $a > 0$. Recall that if $v(x) = \sin \pi l x$, then the exact solution is

$$u(x,t) = e^{-\pi^2 l^2 a t} \sin \pi l x.$$

Consider uniform refinement, that is, for $h = 1/N$ and $k = 1/M$, we let $x_j = jh$ and $t_n = nk$. Write a Matlab program to solve the equation with the finite difference schemes (12.5) and (12.6).

- For $h = 0.05$, $k = 0.05$, $a = .1$ and $v(x) = \sin \pi x$, graph the results at $t = 1$. Which one is better? Give analytical reasons to support your computational results.
- Next, for $h = 0.05$, $k = 0.05$ and $a = 2$, graph the results for $v(x) = \sin 10\pi x$ at $t = .05, .1, .5, 1$. Why are the results so poor? Would a different choice or $r = \frac{k}{h^2}$ improve the results?
- For $v(x) = \sin \pi x + \sin 10\pi x$ and $a = 1$, suppose that we want a numerical solution whose relative error is about 10^{-4}, how do you choose k and h for (12.5) and (12.6)? What if we want the error is about 10^{-6}?