Numerical Partial Differential Equations I
Homework 4

(Due: Jan. 24, 2006)

Consider the hyperbolic partial differential equation

$$\begin{aligned} u_t + cu_x &= 0, \quad -1 < x < 1, \quad 0 < t < .5, \\ u(-1,t) &= b(t), \quad 0 < t < .5, \\ u(x,0) &= v(x), \quad -1 < x < 1, \end{aligned} \tag{1}$$

where $c > 0$. Recall that the exact solution to (1) is given by

$$u(x,t) = \begin{cases} b(t - \frac{x}{c}) & x \leq ct - 1 \\ v(x - ct) & x > ct - 1 \end{cases}$$

Consider uniform refinement, that is, for $h = 1/N$ and $k = 1/M$, we let $x_j = jh$ and $t_n = nk$. Write a Matlab program to solve the equation with the Lax-Friedrichs method (13.5), Lax-Wendroff (13.17) method and Upwind method (13.22, 13.23).

- Consider the problem (1) with $c = 0.5$, $b(t) = 0$, and

$$v(x) = \begin{cases} (x - 0.5)^2 (x + 0.5)^2 \cdot 2^4 & -0.5 \leq x \leq 0.5 \\ 0 & otherwise \end{cases}$$

  Study the performance (stability and accuracy) of the schemes, compare the computed results with the exact solution and discuss the difference.

- Consider the problem (1) with $c = 0.5$, $b(t) = 0$, and

$$v(x) = \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

  Study the performance (stability and accuracy) of the schemes, compare the computed results with the exact solution and discuss the difference.