Calculus II
Ver A

Name:____________________
Student ID:____________________

Quiz 10

Jun. 6, 2007

1. (5 points) For a differentiable function $g(t) = f(x(t), y(t))$ with $f(x, y) = 10x^{1/4}y^{3/4}$, $x(t) = te^t$ and $y(t) = t^2$, use the **chain rule** to find $g'(t)$.

$g'(t) = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

$f(x,y) = 10x^{1/4}y^{3/4}$

$f_x = 10y^{3/4}\cdot\frac{1}{4}\cdot x^{-3/4}$

$f_y = 10x^{1/4}\cdot\frac{3}{4}y^{-1/4}$

$x(t) = te^t,\quad x'(t) = e^t + te^t = (1+t)e^t$

$y(t) = t^2,\quad y'(t) = 2t.$

$g'(t) = \frac{10}{4}x^{-3/4}y^{3/4}(1+t)e^t + \frac{30}{4}x^{1/4}y^{-1/4}(2t)$

$= \frac{5}{2}(te^t)^{-3/4}(t^2)^{3/4}(1+t)e^t + \frac{15}{2}(te^t)^{1/4}(t^2)^{-1/4}$

2. (5 points) Suppose that $g(u, v) = f(x(u, v), y(u, v))$, where f is a differentiable function of x and y and where $x = x(u, v)$ and $y = y(u, v)$ both have first-order partial derivatives. Given that

$x_u(1,2) = 0$,	$y_u(1,2) = 3$,	$x(1,2) = 5$,	$f_x(5,6) = 7$,
$x_v(1,2) = 1$,	$y_v(1,2) = 4$,	$y(1,2) = 6$,	$f_y(5,6) = 8$

use the **chain rule** to find $g_v(1, 2)$.

$\frac{\partial g}{\partial v}(u,v) = \frac{\partial f}{\partial x}(x(u,v), y(u,v))\frac{\partial x}{\partial v}(u,v) + \frac{\partial f}{\partial y}(x(u,v), y(u,v))\frac{\partial y}{\partial v}(u,v)$

$\frac{\partial g}{\partial v}(1,2) = \frac{\partial f}{\partial x}(x(1,2), y(1,2))\frac{\partial x}{\partial v}(1,2) + \frac{\partial f}{\partial y}(x(1,2), y(1,2))\frac{\partial y}{\partial v}(1,2)$

$= \frac{\partial f}{\partial x}(5,6)\cdot 1 + \frac{\partial f}{\partial y}(5,6)\cdot 4$

$= 7\cdot 1 + 8\cdot 4 = 39$

\#

3. (5 pts) Given that $f(x, y) = 2x^2 + y^2 + 2x + 9$, compute the directional derivative of f at $(1, 2)$ in the direction of the vector $< \frac{1}{2}, \frac{\sqrt{3}}{2} >$

$\vec{u} = <\frac{1}{2}, \frac{\sqrt{3}}{2}>,\quad \|\vec{u}\| = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = 1$

$D_{\vec{u}}f = \nabla f\cdot\vec{u},$

$\nabla f = <4x+2, 2y>$

$\nabla f(1,2) = <6, 4>$

$D_{\vec{u}}f(1,2)$

$= <6,4>\cdot<\frac{1}{2}, \frac{\sqrt{3}}{2}>$

$= 3 + 2\sqrt{3}$

\#

4. (5 pts) Find the critical points of $f(x, y) = 2x^2 + y^2 + 2x + 9$.

$\nabla f = <4x+2, 2y>$

$\nabla f = 0 \Rightarrow \begin{cases} 4x+2=0 \\ y=0 \end{cases} \Rightarrow x = -\frac{1}{2},\ y = 0$

critical point $(-\frac{1}{2}, 0)$ \#

extra: $f_{xx} = 4$, $f_{xy} = 0$, $f_{yy} = 2$

$D(-\frac{1}{2}, 0) = \begin{vmatrix} 4 & 0 \\ 0 & 2 \end{vmatrix} = 8 > 0$

$f_{xx}(4) > 0$

$\Rightarrow (-\frac{1}{2}, 0)$ is a local min

In fact, $(-\frac{1}{2}, 0)$ is the abs. min