Calculus II
Ver A

Name:____________________
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Quiz 11

Jun. 13, 2007

1. (10 pts) Find the maximum and minimum values of the function $f(x,y) = 4xy$ subject to the constraint $4x^2 + y^2 \leq 8$

1° Critical points:

$\nabla f = \langle 4y, 4x \rangle \Rightarrow (0,0)$ is the only critical point.

$(4\cdot(0)^2 + 0^2 = 0 \leq 8)$

2° Let $g(x,y) = 4x^2 + y^2 - 8$, $\nabla g = \langle 8x, 2y \rangle$

$$\begin{cases} 4y = 8\lambda x & ① \\ 4x = 2\lambda y & ② \\ 4x^2 + y^2 - 8 = 0 & ③ \end{cases}$$

① $\Rightarrow y = 2\lambda x$ ④

plug ④ into ②

$4x = 4\lambda^2 x \Rightarrow x = 0$ or $\lambda^2 = 1$

case 1 "$x = 0$"

$\Rightarrow y = 0 \Rightarrow 4(0)^2 + (0)^2 - 8 \neq 0$

case 2 $\lambda = 1$

$\Rightarrow y = 2x, \Rightarrow 4x^2 + (2x)^2 - 8 = 0 \Rightarrow x^2 = 1$ or $x = \pm 1$

$\Rightarrow (1, 2), (-1, -2)$

case 3 $\lambda = -1$

$\Rightarrow y = -2x \Rightarrow 4x^2 + (-2x)^2 - 8 = 0 \Rightarrow x^2 = 1$ or $x = \pm 1$

$\Rightarrow (1, -2), (-1, 2)$

3°

$f(0,0) = 0$

$f(1,2) = 8$ ← max

$f(1,-2) = -8$ } min

$f(-1,2) = -8$ }

$f(-1,-2) = 8$ ← max

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2. (10 pts) Evaluate the iterated integral by first changing the order of integration.

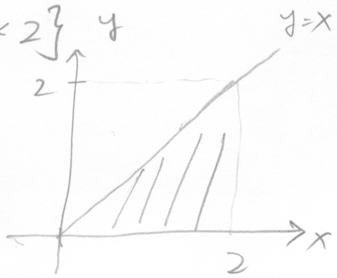
$$\int_0^2 \int_y^2 6xe^{x^3}\,dx\,dy$$

1° region:

$R = \{(x,y) \mid 0 \leq y \leq 2,\ y \leq x \leq 2\}$

rewrite as

$R = \{(x,y) \mid 0 \leq x \leq 2,\ 0 \leq y \leq x\}$



2°

$$\int_0^2 \int_y^2 6xe^{x^3}\,dx\,dy$$

$$= \int_0^2 \int_0^x 6xe^{x^3}\,dy\,dx$$

$$= \int_0^2 6xe^{x^3}\,y\Big|_0^x\,dx$$

$$= \int_0^2 6xe^{x^3}(x-0)\,dx$$

$$= \int_0^2 6x^2e^{x^3}\,dx$$

$$= 2e^{x^3}\Big|_0^2$$

$$= 2e^{2^3} - 2e^{0^3}$$

$$= 2(e^8 - 1)$$

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