Calculus II
Ver A

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Quiz 8

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1. (10 pts) Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{6x^3y}{x^6+y^2}$$

along $x=0$,

$$\lim_{(x,y)\to(0,0)} \frac{6x^3y}{x^6+y^2} = \lim_{y\to 0} \frac{6\cdot 0^3 y}{0+y^2} = 0$$

along $y=x^3$ (分母為 x^6+y^2)

$$\lim_{(x,y)\to(0,0)} \frac{6x^3y}{x^6+y^2} = \lim_{x\to 0} \frac{6x^3\cdot x^3}{x^6+(x^3)^2} = \lim_{x\to 0} \frac{6x^6}{2x^6} = 3 \neq 0$$

$\Rightarrow$ the limit DNE

2. (10 pts) Determine the constant c, such that the function is continuous.

$$f(x,y) = \begin{cases} \frac{x^3y^2}{x^2+y^2} & \text{for } (x,y) \neq (0,0), \\ c & \text{for } (x,y) = (0,0). \end{cases}$$

1° along $x=0$,

$$\lim_{(x,y)\to(0,0)} \frac{x^3y^2}{x^2+y^2} = 0.$$

2° Consider, for $(x,y) \neq 0$

$$|f(x,y) - 0| = \left|\frac{x^3y^2}{x^2+y^2}\right|.$$

For $y \neq 0$, since $x^2+y^2 \geq y^2$, $\frac{1}{x^2+y^2} \leq \frac{1}{y^2}$

$$\Rightarrow |f(x,y)-0| = \left|\frac{x^3y^2}{x^2+y^2}\right| \leq \frac{|x^3|\, y^2}{y^2} = |x^3|$$

For $y=0$ $(x \neq 0)$

$$|f(x,y)-0| = \left|\frac{x^3\cdot 0}{x^2+0^2}\right| = 0 \leq |x^3|$$

Take $g(x,y) = |x^3|$. $\lim_{(x,y)\to(0,0)} |x^3| = 0$.

Since $|f(x,y) - 0| \leq g(x,y) = |x^3|$ for all $(x,y) \neq (0,0)$, & $\lim_{(x,y)\to(0,0)} |x^3| = 0$

by Thm 2.1, $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

3° $\because$ $\frac{x^3y^2}{x^2+y^2}$ is a ration fun,

f is contin for $(x,y) \neq (0,0)$

To make f a continuous, we simply take

$$f(0,0) = \lim_{(x,y)\to(0,0)} f(x,y)$$

or $c = \lim_{(x,y)\to(0,0)} f(x,y) = 0$ #