**Calculus II** Name:______________________

**Ver A** Student ID:______________________

# Quiz 9

May. 30, 2007

1. (10 pts) Given $f(x,y) = x^3 - 3xy + y^3$, find the partial derivatives, $f_x$, $f_y$, $f_{xy}$ and $f_{xx}$.

$f_x = \frac{\partial}{\partial x}(x^3 - 3xy + y^3) = \frac{\partial}{\partial x}(x^3) - 3y\frac{\partial}{\partial x}(x) + \frac{\partial}{\partial x}(y^3)$
$= 3x^2 - 3y$

$f_y = \frac{\partial}{\partial y}(x^3 - 3xy + y^3) = 0 - 3x\frac{\partial}{\partial y}(y) + \frac{\partial}{\partial y}(y^3)$
$= -3x + 3y^2$

$f_{xy} = \frac{\partial}{\partial y}(f_x) = \frac{\partial}{\partial y}(3x^2 - 3y) = -3$

$f_{xx} = \frac{\partial}{\partial x} f_x = \frac{\partial}{\partial x}(3x^2 - 3y) = 6x$

2. (10 pts) Find the equation of the tangent plane to the surface at the given point.

$$z = \underbrace{x^2 - y^2 + 1}_{f(x,y)} \quad \text{at } (2,1,4)$$

$f_x = 2x$, $f_x(2,1) = 4$

$f_y = -2y$, $f_y(2,1) = -2$

normal vector to the tangent plane

$\vec{n} \parallel \langle 0, 1, f_y(2,1)\rangle \times \langle 1, 0, f_x(2,1)\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -2 \\ 1 & 0 & 4 \end{vmatrix}$

$= \langle 4, -2, -1\rangle$

Take $\vec{n} = \langle 4, -2, -1\rangle$, let $(x, y, z)$ be a point on the tangent plane, then we have

$\langle x-2, y-1, z-4\rangle \cdot \langle 4, -2, -1\rangle = 0$

or $4(x-2) - 2(y-1) - (z-4) = 0$

or $z = 4(x-2) - 2(y-1) + 4$ #