- Theorem 10.5.1 The Chain Rule (I)
  If $z = f(x(t), y(t))$, where $x(t)$ and $y(t)$ are differentiable and $f(x,y)$ is a differentiable function of x and y, then
  $$\frac{dz}{dt} = \frac{d}{dt}[f(x(t), y(t))] = \frac{\partial f}{\partial x}(x(t), y(t))\frac{dx}{dt} + \frac{\partial f}{\partial y}(x(t), y(t))\frac{dy}{dt}.$$

- Theorem 10.5.2 Chain Rule (II)
  Suppose that $z = f(x,y)$, where f is a differentiable function of x and y and where $x = x(s,t)$ and $y = y(s,t)$ both have first-order partial derivatives. Then we have the chain rules:
  $$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$
  $$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}.$$

- Theorem 10.7.2 Second Derivatives Test
  Suppose that $f(x,y)$ has continuous second-order partial derivatives in some open disk containing the point (a,b) and that $f_x(a,b) = f_y(a,b) = 0$. Define the **discriminant** D for the point (a,b) by
  $$D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [\, f_{xy}(a,b)]^2.$$
  (1) If $D(a,b) > 0$ and $f_{xx}(a,b) > 0$, then f has a local minimum at (a,b).
  (2) If $D(a,b) > 0$ and $f_{xx}(a,b) < 0$, then f has a local maximum at (a,b).
  (3) If $D(a,b) < 0$, then f has a saddle point at (a,b).
  (4) If $D(a,b) = 0$, then no conclusion can be drawn.

- Theorem 10.7.3 Extreme Value Theorem (I)
  Suppose that $f(x,y)$ is continuous on the closed and bounded region $R \subset \mathbb{R}^2$. Then f has both an absolute maximum and an absolute minimum on R. Further, the absolute extrema must occur at either a critical point in R or on the boundary of R.

- Theorem 10.8.1 Method of Lagrange Multiplier
  Suppose that $f(x,y)$ and $g(x,y)$ are functions with continuous first partial derivatives and $\nabla g(x,y) \neq \mathbf{0}$ on the surface $g(x,y) = 0$. Suppose that either
  (1) the minimum value of $f(x,y)$ subject to the constraint $g(x,y) = 0$ occurs at (x_0, y_0); or
  (2) the maximum value of $f(x,y)$ subject to the constraint $g(x,y) = 0$ occurs at (x_0, y_0).
  Then $\nabla f(x_0, y_0, z_0) = \lambda \nabla g(x_0, y_0, z_0)$ for some constant λ.

- Definition 11.8.1 Jacobian of a transformation
  $$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial u} \end{vmatrix}$$

- Theorem 11.8.1
  Suppose that the region S in the uv-plane is mapped onto the region R in the xy-plane by the one-to-one transformation T defined by $x = g(u,v)$ and $y = h(u,v)$, where g and h have continuous first partial derivatives on S. If f is continuous on R and the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ is nonzero on S, then
  $$\iint_R f(x,y)dA = \iint_S f(g(u,v), h(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv.$$