

- Taylor Polynomial

Given a smooth function $f(x)$. The Taylor Series expansion for f is

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(c)}{k!} (x - c)^k$$

The Taylor Polynomial of degree n , denoted by $P_n(x)$, expanded about $x = c$ is given by

$$P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(c)}{k!} (x - c)^k$$

- Taylor's Theorem Suppose that f has derivatives of all orders in the interval $(c - r, c + r)$, for some $r > 0$ and that $\lim_{n \rightarrow \infty} R_n(x) = 0$, for all x in $(c - r, c + r)$. Then, the Taylor series for f expanded about $x = c$ converges to $f(x)$, that is,

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(c)}{k!} (x - c)^k,$$

for all x in $(c - r, c + r)$.

Test	When to use	Conclusions
Geometric Series	$\sum_{k=0}^{\infty} ar^k$	Converges to $\frac{a}{1-r}$ if $ r < 1$; diverges if $ r \geq 1$.
k th-Term Test	All series	If $\lim_{k \rightarrow \infty} a_k \neq 0$, the series diverges.
Integral Test	$\sum_{k=1}^{\infty} a_k$ where $f(k) = a_k$ and f is continuous, decreasing and $f(x) \geq 0$	$\sum_{k=1}^{\infty} a_k$ and $\int_1^{\infty} f(x) dx$ both converge or both diverge.
p -series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	Converges for $p > 1$; diverges for $p \leq 1$.
Comparison Test	$\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$, where $0 \leq a_k \leq b_k$	If $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ converges. If $\sum_{k=1}^{\infty} a_k$ diverges, then $\sum_{k=1}^{\infty} b_k$ diverges.
Limit Comparison Test	$\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$, where $a_k, b_k > 0$ and $\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = L > 0$	$\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ both converge or both diverge.
Alternating Series Test	$\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ where $a_k > 0$ for all k	If $\lim_{k \rightarrow \infty} a_k = 0$ and $a_{k+1} \leq a_k$ for all k , then the series converges.
Absolute Convergence	Series with some positive and some negative terms (including alternating series)	If $\sum_{k=1}^{\infty} a_k $ converges, then $\sum_{k=1}^{\infty} a_k$ converges (absolutely).
Ratio Test	Any series (especially those involving exponentials and/or factorials)	For $\lim_{k \rightarrow \infty} \left \frac{a_{k+1}}{a_k} \right = L$, if $L < 1$, $\sum_{k=1}^{\infty} a_k$ converges absolutely if $L > 1$, $\sum_{k=1}^{\infty} a_k$ diverges, if $L = 1$, no conclusion.