Calculus II — Midterm Practice problems
Time: 4/30(M), 3:10-5:00; Place: 數學系 3173, 3174 (1F)

Chap 4: Sec. 4.10.

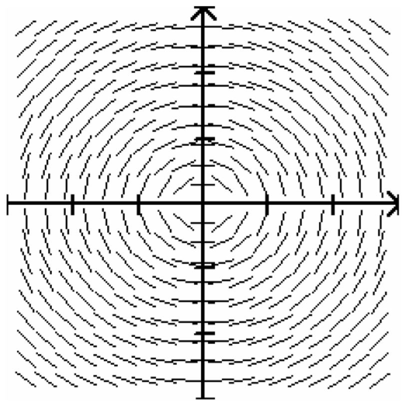
1. Evaluate the given integral
   i) $\int_{-\infty}^{\infty} \frac{e^{-x}}{(1+e^{-x})^2}\,dx$ $\int_{-\infty}^{\infty} x^3\,dx$ iii) $\lim_{R\to\infty}\int_{-R}^{R} x^3\,dx$
2. Determine whether the integral converges or diverges:
   i) $\int_0^1 x^{-1/3}\,dx$ ii) $\int_0^1 x^{-4/3}\,dx$
   iii) $\int_1^{\infty} x^{-1/3}\,dx$ iv) $\int_{-1}^{1} x^{-1/3}\,dx$

Chap 5: Sec. 5.3-Sec. 5.4.

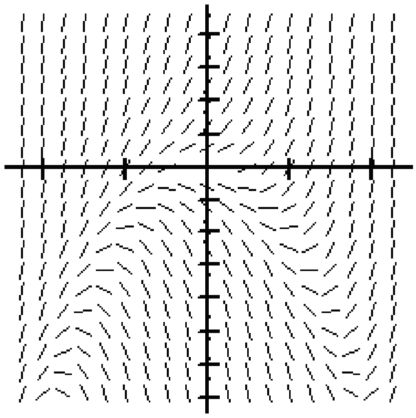
1. Set up a definite integral for the arc length of an ellipse $x^2+4y^2=4$.
2. Set up the integral for the surface area of the surface of revolution. $y=e^x, 0\le x\le 1$, revolved about x-axis.
3. An object is released from a height of 10m with an upward velocity of $5m/s$. Let $y(t)$ be the height of the object. Identify the initial conditions $y(0)$ and $y'(0)$. Find $y(t)$.

Chap 6: Sec. 6.1-Sec. 6.3.

1. Two years ago, there were 4 grams of a radioactive substance . Now there are 3 grams. How much was there 10 years ago?
2. Find the size of permanent endowment needed to generate an annual \$2,000 forever at 10% (annual) interest compounded continuously.
3. Select the differential equation which corresponds to the direction field below.
   (a) $y'=-\frac{1}{2y}$
   (b) $y'=-\frac{2y}{x}$,
   (c) $y'=-\frac{2x}{y}$
   (d) $y'=-\frac{1}{y^2}$



4. Select the differential equation which corresponds to the direction field below.
   (a) $y'=-xy$
   (b) $y'=y+x^2$,
   (c) $y'=y^2+x^3$
   (d) $y'=-x-y^2$



5. Solve the IVP:
   i) $y'=\frac{x-1}{y^2}, y(0)=2$; ii) $y'=\frac{x-1}{y}, y(0)=-2$.
6. Solve the initial value problem
   i) $y'(t)=y(t)$, $y(0)=0$; ii) $y'(t)=(y(t)-4)$, $y(0)=4$.

Chap 7. Sec. 7.1-Sec. 7.8.

1. Determine the convergence of a sequence. If it converges, find the limit of the sequence.
   A) $a_n = \frac{3n^2+n-2}{4n^2-n}$ B) $a_n = \frac{\cos n}{n^2}$
   C) $a_n = 1$ D) $a_n = (-1)^n$

2. Determine if the series converges or diverges. If it converges, find the sum of the series.
   i) $\sum_{k=0}^{\infty}(\frac{1}{3})^k$ ii) $\sum_{k=0}^{\infty}\frac{k+1}{k^2-4}$
   iii) $\sum_{k=2}^{\infty}\frac{1}{k(k-1)}$

3. Determine if the series converges or diverges.
   i) $\sum_{k=0}^{\infty}\frac{3}{\sqrt[3]{k}}$ ii) $\sum_{k=2}^{\infty}\frac{1}{k(k-1)}$
   iii) $\sum_{k=0}^{\infty}((-1)^k - (\frac{1}{3})^k)$ iv) $\sum_{k=0}^{\infty}\frac{k+1}{k^3-4}$
   v)$\sum_{k=2}^{\infty}\frac{\cos k+1}{k^2}$

4. Use the Integral Test to test the convergence of the series $\sum_{k=2}^{\infty}\frac{1}{k(\ln k)^3}$

5. If $a_k = \begin{cases} 1/k & \text{if k is odd} \\ 1/k^2 & \text{if k is even} \end{cases}$, determine the convergence of the series, $\sum_{k=1}^{\infty}(-1)^k a_k$.

6. Determine if the series is absolutely convergent, conditionally convergent or divergent.
   i) $\sum_{k=1}^{\infty}(-1)^k\frac{k}{k^3+1}$ ii) $\sum_{k=1}^{\infty}(-1)^k\frac{\sqrt{k}}{k+1}$
   iii) $\sum_{k=1}^{\infty}(-1)^k\frac{3^k}{(2k)!}$ iv) $\sum_{k=1}^{\infty}(-1)^k\frac{1}{(2k+1)}$

7. Determine the radius and interval of convergence of the power series.
   i) $\sum_{k=1}^{\infty}\frac{k}{2^k}(x-2)^k$ ii) $\sum_{k=1}^{\infty}(x-3)^k$
   iii) $\sum_{k=1}^{\infty}k!(x-2)^k$ iv) $\sum_{k=1}^{\infty}\frac{2^k}{k!}(x-3)^k$

8. For $f(x) = e^x$, find the Taylor polynomial of degree 2 expanded about $c = 1$.

9. Find the Taylor series of e^x, e^{-x^2} and $x^2e^{-x^2}$ about $c = 0$. Determine the corresponding radius and interval of convergence.

10. Given that $\frac{1}{1+x} = \sum_{k=0}^{\infty}(-1)^k x^k$, for $-1 < x < 1$, find the Taylor series of
    i) $\ln(1+x)$ ii) $\frac{1}{1+x^2}$.
    iii) $x\ln(1+x)$ iv) $\frac{x^2}{1+x^2}$.
    Determine the corresponding radius and interval of convergence.

11. $\sum_{k=0}^{\infty}(-1)^k\frac{1}{k+1}$ =?