Calculus II **Time: 4/30(M), 3:10-5:00;**

Midterm Practice problems **Place: 數學系 3173, 3174 (1F)**

Chap 4: Sec. 4.10.

1. Evaluate the given integral
   i) $\int_{-\infty}^{\infty} \frac{e^{-x}}{(1+e^{-x})^2}\,dx$ $\int_{-\infty}^{\infty} x^3\,dx$ iii) $\lim_{R\to\infty} \int_{-R}^{R} x^3\,dx$
   Ans: (i) 1 (ii) DIV (iii) 0

2. Determine whether the integral converges or diverges:
   i) $\int_0^1 x^{-1/3}\,dx$ ii) $\int_0^1 x^{-4/3}\,dx$
   iii) $\int_1^{\infty} x^{-1/3}\,dx$ iv) $\int_{-1}^{1} x^{-1/3}\,dx$ Ans: (i) 3/2
   (ii) DIV (iii) DIV (iv) 0

Chap 5: Sec. 5.3-Sec. 5.4.

1. Set up a definite integral for the arc length of an ellipse $x^2 + 4y^2 = 4$. Ans: $4\int_0^2 \sqrt{1 + (-\frac{x}{4\sqrt{1-x^2/4}})^2}\,dx$ or $4\int_0^{\pi/2} \sqrt{(-2\sin\theta)^2 + (\cos\theta)^2}\,d\theta$

2. Set up the integral for the surface area of the surface of revolution. $y = e^x, 0 \le x \le 1$, revolved about x-axis. Ans: $S = \int_0^1 2\pi e^x \sqrt{1 + (e^x)^2}\,dx$

3. An object is released from a height of 10m with an upward velocity of $5m/s$. Let $y(t)$ be the height of the object. Identify the initial conditions $y(0)$ and $y'(0)$. Find $y(t)$. Ans: $y(0) = 10$;$y'(0) = 5$;$y(t) = \frac{-}{1}2gt^2 + 5t + 10$;

Chap 6: Sec. 6.1-Sec. 6.3.

1. Two years ago, there were 4 grams of a radioactive substance . Now there are 3 grams. How much was there 10 years ago?
   Ans: $\frac{4^5}{3^4}$

2. Find the size of permanent endowment needed to generate an annual \$2,000 forever at 10% (annual) interest compounded continuously. Ans: $2000 \cdot e^{-0.1}$

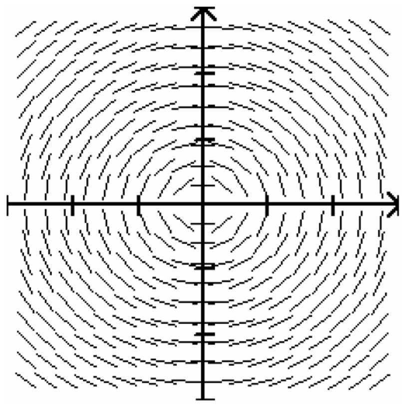
3. Select the differential equation which corresponds to the direction field below.

   (a) $y' = -\frac{1}{2y}$

   (b) $y' = -\frac{2y}{x}$,

   (c) $\underline{y' = -\frac{2x}{y}}$

   (d) $y' = -\frac{1}{y^2}$



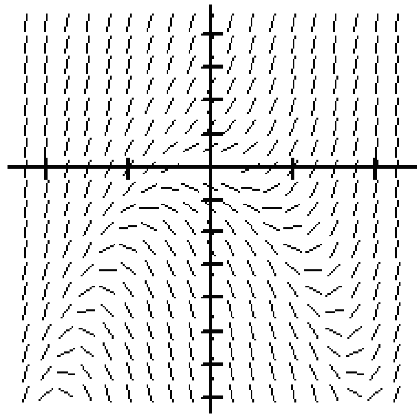
4. Select the differential equation which corresponds to the direction field below.

   (a) $y' = -xy$

   (b) $\underline{y' = y + x^2}$,

   (c) $y' = y^2 + x^3$

   (d) $y' = -x - y^2$



5. Solve the IVP:
   i) $y' = \frac{x-1}{y^2}, y(0) = 2$; ii) $y' = \frac{x-1}{y}, y(0) = -2$.

   Ans: (i) $y = (\frac{3}{2}x^2 - 3x + 8)^{1/3}$;(ii) $y(t) = -\sqrt{x^2 - 2x + 4}$

6. Solve the initial value problem
   i) $y'(t) = y(t)$, $y(0) = 0$; ii) $y'(t) = (y(t) - 4)$, $y(0) = 4$.

   Ans: (i) $y(t0 = 0$;(ii) $y(t) = 4$

Chap 7. Sec. 7.1-Sec. 7.8.

1. Determine the convergence of a sequence. If it converges, find the limit of the sequence.
   A) $a_n = \frac{3n^2+n-2}{4n^2-n}$ B) $a_n = \frac{\cos n}{n^2}$
   C) $a_n = 1$ D) $a_n = (-1)^n$

   Ans: A) $\frac{3}{4}$; B) 0; C) 1; D) Div

2. Determine if the series converges or diverges. If it converges, find the sum of the series.
   i) $\sum_{k=0}^{\infty}(\frac{1}{3})^k$ ii) $\sum_{k=0}^{\infty}\frac{k+1}{k^2-4}$
   iii) $\sum_{k=2}^{\infty}\frac{1}{k(k-1)}$

   Hint/Ans: i) Geometric Series; ii) DIV; Limit comparison Test (Compared with Harmonic Series); iii) Telescope Series; example 2.3.

3. Determine if the series converges or diverges.
   i) $\sum_{k=0}^{\infty}\frac{3}{\sqrt[3]{k}}$ ii) $\sum_{k=2}^{\infty}\frac{1}{k(k-1)}$
   iii) $\sum_{k=0}^{\infty}((-1)^k - (\frac{1}{3})^k)$ iv) $\sum_{k=0}^{\infty}\frac{k+1}{k^3-4}$
   v)$\sum_{k=2}^{\infty}\frac{\cos k+1}{k^2}$

   Hint/Ans:i) Div (p-series) ;ii) Conv (Telescope Series); iii)Div (Thm 2.3); iv) Conv (Limit Comparison Test, p-series) ;v)Conv (Comparison Test)

4. Use the Integral Test to test the convergence of the series $\sum_{k=2}^{\infty}\frac{1}{k(\ln k)^3}$

   Hint: Substitution $s = \ln x$

5. If $a_k = \begin{cases} 1/k & \text{if k is odd} \\ 1/k^2 & \text{if k is even} \end{cases}$, determine the convergence of the series, $\sum_{k=1}^{\infty}(-1)^k a_k$.

   Hint: Div (Thm 2.3)

6. Determine if the series is absolutely convergent, conditionally convergent or divergent.
   i) $\sum_{k=1}^{\infty}(-1)^k\frac{k}{k^3+1}$ ii) $\sum_{k=1}^{\infty}(-1)^k\frac{\sqrt{k}}{k+1}$
   iii) $\sum_{k=1}^{\infty}(-1)^k\frac{3^k}{(2k)!}$ iv) $\sum_{k=1}^{\infty}(-1)^k\frac{1}{(2k+1)}$

   Hint/Ans: i)Abs Conv (Limit Comparison Test, p-series);ii) Conditionally Conv (Limit Comparison Test, p-series, Alternating Series Test);
   iii) Abs Conv (Ratio Test);iv) Conditionally Conv (Limit Comparison Test, p-series, Alternating Series Test)

7. Determine the radius and interval of convergence of the power series.
   i) $\sum_{k=1}^{\infty}\frac{k}{2^k}(x-2)^k$ ii) $\sum_{k=1}^{\infty}(x-3)^k$
   iii) $\sum_{k=1}^{\infty}k!(x-2)^k$ iv) $\sum_{k=1}^{\infty}\frac{2^k}{k!}(x-3)^k$

   Ans: i) $(0, 4)$, $r = 2$;ii) $(2, 4)$, $r = 1$;iii)$\{2\}$,$r = 0$;iv) $(-\infty, \infty)$,$r = \infty$

8. For $f(x) = e^x$, find the Taylor polynomial of degree 2 expanded about $c = 1$.

   Ans: $P_2(x) = e + \frac{e}{1}(x-1) + \frac{e^2}{2}(x-1)^2$

9. Find the Taylor series of e^x, e^{-x^2} and $x^2e^{-x^2}$ about $c = 0$. Determine the corresponding radius and interval of convergence.

   Ans: $e^x = \sum_{k=0}^{\infty}\frac{1}{k!}x^k$, for $x \in (-\infty, \infty)$, $r = \infty$;
   $e^{-x^2} = \sum_{k=0}^{\infty}(-1)^k\frac{1}{k!}x^{2k}$, for $x \in (-\infty, \infty)$, $r = \infty$;
   $x^2e^{-x^2} = \sum_{k=0}^{\infty}(-1)^k\frac{1}{k!}x^{2k+2}$, for $x \in (-\infty, \infty)$, $r = \infty$

10. Given that $\frac{1}{1+x} = \sum_{k=0}^{\infty}(-1)^k x^k, \quad \text{for } -1 < x < 1$, find the Taylor series of

i) $\ln(1+x)$

ii) $\frac{1}{1+x^2}$.

iii) $x\ln(1+x)$

iv) $\frac{x^2}{1+x^2}$.

Determine the corresponding radius and interval of convergence.

Ans: i)$\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} x^{k+1}, \quad \text{for } -1 < x \le 1,\ r = 1;$

ii)$\frac{1}{1+x^2} = \sum_{k=0}^{\infty}(-1)^k x^{2k}, \quad \text{for } -1 < x < 1,,\ r = 1;$

iii)$x\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} x^{k+2}, \quad \text{for } -1 < x \le 1,\ r = 1$

iv)$\frac{x^2}{1+x^2} = \sum_{k=0}^{\infty}(-1)^k x^{2k+2}, \quad \text{for } -1 < x < 1,,\ r = 1;$

11. $\sum_{k=0}^{\infty}(-1)^k \frac{1}{k+1} = ?$

Ans: $\ln 2$