Calculus I Midterm 2 Practice Problems

1. Use the linear approximation of the function $f(x) = (x+1)^{1/4}$ to estimate $(1.02)^{1/4}$.
2. Two hallways, one 8 feet wide and the other 1 feet wide, meet at right angles. Determine the length of the longest ladder that can be carried horizontally from one hallway to the other.
3. Find the point on the curve $y = x^2$ closest to the point $(0, 1)$
4. Suppose a wire 4 ft long is to be cut into two pieces. One will be formed into a square and the other one will be formed into a regular triangle. Find the size of each piece to minimize the total area of the two region.
5. Suppose a 6-ft tall person is 12 ft away from an 18-ft tall lamppost. If the person is moving away from the lamppost at a rate of 2 ft/s, at what rate is the length of the shadow changing?
6. Parametric equations for the position of an object is given. Find the object's velocity and speed at the given times, and describe its motion.
$$\begin{cases} x = 2\cos 2t + \sin 5t \\ y = 2\sin 2t + \cos 5t \end{cases} \quad (a) t = 0, \quad (b) t = \frac{\pi}{2}$$
7. Find the derivatice of the function
$$f(x) = \int_x^{\cos x} \sqrt{1-t^2}\, dt$$
8. If $x \sin x = \int_0^{x^2} f(t)\, dt$, where f is a continuous function, find $f(4)$.
9. $\int \frac{1}{x^2-4x+3}\, dx$
10. $\int \frac{1}{x^2-4x+5}\, dx$
11. $\int \frac{\ln x}{x}\, dx$
12. $\int e^{2x} \sin 2x\, dx$
13. $\int_0^2 \frac{x^2}{(x^2+4)^2}\, dx$
14. $\int \frac{1}{\sqrt{x}+x}\, dx$
15. $\int \tan^4 x \sec^4 x\, dx$
16. $\int \tan x\, dx$
17. $\int \sec x\, dx$
18. $\int_{-2}^{1} |2x+1|\, dx$
19. $\int_0^1 x^{-1/3}\, dx$
20. $\int_1^\infty x^{-1/3}\, dx$
21. $\int_0^\infty \cos x\, dx$
22. $\int_1^\infty \frac{\sin x + 2}{x}\, dx$
23. Find the average value of $f(x) = \sqrt{x}$ on the interval $[0, 9]$
24. Find the area of the region bounded by $y = 2\cos x$, $y = \sin 2x$ for $x \in [-\pi, \pi]$
25. Let Ω be the region bounded by $y = \sec x$, $x = 0$, $x = \frac{\pi}{4}$ and $y = 0$. Find integrals represent the volume of the solids generated by Ω about (a) x-axis, (b) y-axis, (c) $y = -1$, (d) $x = -1$.

- Double-Angle

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

- Derivative formulas

$$\begin{aligned}
\tfrac{d}{dx}\sin^{-1}x &= \frac{1}{\sqrt{1-x^2}}, \\
\tfrac{d}{dx}\cos^{-1}x &= -\frac{1}{\sqrt{1-x^2}}, \\
\tfrac{d}{dx}\tan^{-1}x &= \frac{1}{1+x^2}, \\
\tfrac{d}{dx}\cot^{-1}x &= -\frac{1}{1-x^2}, \\
\tfrac{d}{dx}\sec^{-1}x &= \frac{1}{|x|\sqrt{x^2-1}}, \\
\tfrac{d}{dx}\csc^{-1}x &= -\frac{1}{|x|\sqrt{x^2-1}}
\end{aligned}$$