Calculus I Midterm 2 Practice Problems

1. Use the linear approximation of the function $f(x) = (x+1)^{1/4}$ to estimate $(1.02)^{1/4}$.

$$L(x) = 1 + \frac{1}{4}(x-0), \quad (1.02)^{1/4} \approx L(0.02) = 1.005$$

2. Two hallways, one 8 feet wide and the other 1 feet wide, meet at right angles. Determine the length of the longest ladder that can be carried horizontally from one hallway to the other.
   Find the minimum of $f(x) = (\frac{8}{x}+1)^2(x^2+1^2)$

3. Find the point on the curve $y = x^2$ closest to the point $(0,1)$
   Find the minimum of $f(x) = (x-0)^2 + (x^2-1)^2$

4. Suppose a wire 4 ft long is to be cut into two pieces. One will be formed into a square and the other one will be formed into a regular triangle. Find the size of each piece to minimize the total area of the two region.
   Find the minimum of $A(x) = (4-x)^2 + \frac{x}{2}\frac{\sqrt{3}}{2}x$ for $x \geq 0$

5. Suppose a 6-ft tall person is 12 ft away from an 18-ft tall lamppost. If the person is moving away from the lamppost at a rate of 2 ft/s, at what rate is the length of the shadow changing?
   Sec 3.8: exercise 23.

6. Parametric equations for the position of an object is given. Find the object's velocity and speed at the given times, and describe its motion.

$$\begin{cases} x = 2\cos 2t + \sin 5t \\ y = 2\sin 2t + \cos 5t \end{cases} \quad (a) t = 0, \quad (b) t = \frac{\pi}{2}$$

   Sec 3.8: exercise 29

7. Find the derivatice of the function

$$f(x) = \int_x^{\cos x} \sqrt{1-t^2}\,dt$$

   Hint: Let $g(x) = \int_0^x \sqrt{1-t^2}\,dt$, $f(x) = g(\cos x) - g(x)$.

8. If $x \sin x = \int_0^{x^2} f(t)\,dt$, where f is a continuous function, find $f(4)$. Hint: Differentiate the equation with respect to x.

9. $\int \frac{1}{x^2-4x+3}\,dx$

$$\frac{1}{2}(\ln|x-3| - \ln|x-1|) + C$$

10. $\int \frac{1}{x^2-4x+5}\,dx$

$$\tan^{-1}(x-2) + C$$

11. $\int \frac{\ln x}{x}\,dx$

$$\frac{1}{2}(\ln x)^2 + C$$

12. $\int e^{2x} \sin 2x\,dx$

$$\frac{1}{4}e^{2x}(\sin 2x - \cos 2x) + C$$

13. $\int_0^2 \frac{x^2}{(x^2+4)^2}\, dx$
Hint: Let $x = 2\tan\theta$, change the terms in the integral to sin and cos, and use double-angle formula.

14. $\int \frac{1}{\sqrt{x}+x}\, dx$
Hint: Let $u = \sqrt{x}$, $u^2 = x$.

15. $\int \tan^4 x \sec^4 x\, dx$
Hint: $u = \tan\theta$

16. $\int \tan x\, dx$
$= \int \frac{\sin x}{\cos x}\, dx = -\ln|\cos x| + C$

17. $\int \sec x\, dx$
Hint: multiply with $\frac{\sec x + \tan x}{\sec x + \tan x}$
$= \ln|\sec x + \tan x| + C$

18. $\int_{-2}^{1} |2x+1|\, dx$
$= \int_{-2}^{-1/2} -2x - 1\, dx + \int_{-1/2}^{1} 2x + 1\, dx$

19. $\int_0^1 x^{-1/3}\, dx$
$= \frac{3}{2}$

20. $\int_1^\infty x^{-1/3}\, dx$
DIV

21. $\int_0^\infty \cos x\, dx$
DIV

22. $\int_1^\infty \frac{\sin x + 2}{x}\, dx$
DIV

23. Find the average value of $f(x) = \sqrt{x}$ on the interval $[0, 9]$
$= \frac{1}{9-0} \int_0^9 \sqrt{x}\, dx = \frac{1}{9}\frac{2}{3} x^{3/2}|_0^9 = 2$

24. Find the area of the region bounded by $y = 2\cos x$, $y = \sin 2x$ for $x \in [-\pi, \pi]$
$A = \int_{-\pi}^{-\pi/2} \sin 2x - 2\cos x\, dx + \int_{-\pi/2}^{\pi/2} 2\cos x - \sin 2x\, dx + \int_{\pi/2}^{\pi} \sin 2x - 2\cos x\, dx$

25. Let Ω be the region bounded by $y = \sec x$, $x = 0$, $x = \frac{\pi}{4}$ and $y = 0$. Find integrals represent the volume of the solids generated by Ω about (a) x-axis, (b) y-axis, (c) $y = -1$, (d) $x = -1$.
(a) $\int_0^{\pi/4} \pi(\sec x)^2\, dx$
(b) $\int_0^{\pi/4} 2\pi x \sec x\, dx$
(c) $\int_0^{\pi/4} \pi[(\sec x + 1)^2 - 1^2]\, dx$
(d) $\int_0^{\pi/4} 2\pi(x+1)\sec x\, dx$

- Double-Angle

$$\sin 2\theta = 2 \sin\theta \cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

- Derivative formulas

$$\begin{aligned}
\tfrac{d}{dx}\sin^{-1}x &= \frac{1}{\sqrt{1-x^2}}, \\
\tfrac{d}{dx}\cos^{-1}x &= -\frac{1}{\sqrt{1-x^2}}, \\
\tfrac{d}{dx}\tan^{-1}x &= \frac{1}{1+x^2}, \\
\tfrac{d}{dx}\cot^{-1}x &= -\frac{1}{1-x^2}, \\
\tfrac{d}{dx}\sec^{-1}x &= \frac{1}{|x|\sqrt{x^2-1}}, \\
\tfrac{d}{dx}\csc^{-1}x &= -\frac{1}{|x|\sqrt{x^2-1}}
\end{aligned}$$