Final of Calculus 1/15/2007

1. **Show your work to get credits!**
2. **No Calculator!**
3. **No Cheating!**

(1) Find the derivatives
(a) $f(x) = \ln(x\sqrt{x^2+1})$, (b) $f(x) = 3^{x^2+x}$.

(2) Find the indefinite integral
(a) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ (b) $\int (2x+1)^5 dx$
(c) $\int \frac{1}{x \ln x} dx$ (d) $\int \frac{x}{\sqrt{3x+1}} dx$.

(3) Find the definite integral
(a) $\int_0^1 xe^{x^2} dx$ (b) $\int_1^2 \frac{1+\ln x}{x} dx$.

(4) Evaluate $\int_0^3 |x-1| dx$

(5) Find the area of the region bounded by the two graphs of functions $f(x) = (x-1)^3$ and $g(x) = x - 1$.

(6) Find the consumer and producer surpluses if the demand function is given by $p_1(x) = 100 - x^2$ and the supply function is given by $p_2(x) = 70 + x$.

(7) The upper half of the ellipse

$$16x^2 + 25y^2 = 400$$

is revolved about the x-axis to form a football like spheroid. Find the volume of the spheroid.

(8) The probability of recall in an experiment is found to be

$$P(a \le x \le b) = \int_a^b \frac{105}{16} x^2 \sqrt{1-x} dx,$$

where x represents the percent of recall. $(0 \le x \le 1)$
Find the probablity that a randomly chosen individual will recall 80% of the material.

(9) Sketch the graph of the function

$$f(x) = \frac{x^3}{x^3 - 1}.$$

Find the intercepts, relative extrema, points of inflection, and asymptotes if they exist.