Final of Calculus 1/15/2007

(1) Find the derivatives
(a) $f(x) = \ln(x\sqrt{x^2+1})$, (b) $f(x) = 3^{x^2+x}$.
Ans:(a) $\frac{1}{x} + \frac{x}{x^2+1}$; (b) $3^{x^2+x}(2x+1)\ln 3$

(2) Find the indefinite integral
(a) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ (b) $\int (2x+1)^5 dx$
(c)$\int \frac{1}{x \ln x} dx$ (d) $\int \frac{x}{\sqrt{3x+1}} dx$.
Ans: (a)$\frac{2}{3}x^{3/2} + 2x^{1/2} + C$; (b) $\frac{1}{12}(2x+1)^6 + C$; (c) $\ln(\ln x) + C$;
(d)$\frac{2}{27}(3x+1)^{3/2} - \frac{2}{9}(3x+1)^{1/2} + C$

(3) Find the definite integral
(a) $\int_0^1 xe^{x^2} dx$ (b) $\int_1^2 \frac{1+\ln x}{x} dx$.
Ans: (a)$\frac{1}{2}e^{x^2}|_0^1 = \frac{1}{2}(e-1)$;(b) $\frac{1}{2}(1+\ln x)^2|_1^2 = \frac{1}{2}((1+\ln 2)^2 - 1^2)$

(4) Evaluate $\int_0^3 |x-1| dx$
Ans: $= \int_0^1 -(x-1)dx + \int_1^3 (x-1)dx = (x - \frac{1}{2}x^2)|_0^1 + (\frac{1}{2}x^2 - x)|_1^3 = \frac{5}{2}$

(5) Find the area of the region bounded by the two graphs of functions $f(x) = (x-1)^3$ and $g(x) = x - 1$.
Ans: $\int_0^1 ((x-3)^3 - (x-1))\, dx + \int_1^2 ((x-3) - (x-1)^3)\, dx = \ldots$

(6) Find the consumer and producer surpluses if the demand function is given by $p_1(x) = 100 - x^2$ and the supply function is given by $p_2(x) = 70 + x$.
Note: Skip this one.

(7) The upper half of the ellipse

$$16x^2 + 25y^2 = 400$$

is revolved about the x-axis to form a football like spheroid. Find the volume of the spheroid.
Ans: $\int_{-5}^5 \pi(\sqrt{16 - \frac{16}{25}x^2})^2\, dx = \ldots$

(8) The probability of recall in an experiment is found to be

$$P(a \le x \le b) = \int_a^b \frac{105}{16} x^2 \sqrt{1-x} dx,$$

where x represents the percent of recall. $(0 \le x \le 1)$
Find the probablity that a randomly chosen individual will recall 80% of the material.
Note: Skip this one (題意不清).

(9) Sketch the graph of the function

$$f(x) = \frac{x^3}{x^3 - 1}.$$

Find the intercepts, relative extrema, points of inflection, and asymptotes if they exist.
Ans: x-intercept: $(0,0)$, y-intercept: $(0,0)$; No relative extrema;
inflection point: $(0,0)$; H-asymptote: $y = 1$; V-asymptoe: $x = 1$