Calculus I Final

1. (a) Let $f(x) = \begin{cases} cx - 2 & \text{for } x \leq c; \\ x^2 + 2x & \text{for } x > c \end{cases}$ Find c such that $f(x)$ is continuous.

   (b) Compute **(without the use of L'Hopital's rule)**

$$\lim_{x \to 0} \frac{\sqrt{x+9} - 3}{x}$$

2. Find the derivatives of the following functions. It is not necessary to simplify your answer:

   (a)

$$f(x) = \sqrt{\frac{(3x+1)^2(5x-1)^3}{(5x+2)^4}}$$

   .

   (b)

$$g(x) = x^{4x}$$

   (c)

$$G(x) = \int_{x^3}^{2x} \sqrt{1+t^4}\, dt$$

3. Compute (a)

$$\lim_{x \to 0} \frac{1 - \cos 4x}{4x^2}$$

   (b)

$$\lim_{x \to 0^+} (1 + \sin x)^{1/x}$$

4. () Find the asymptotes of

$$f(x) = \frac{(x-2)^2}{x^2 - 4}$$

   .

5. An automobile dealer is selling cars at a price of \$16,000. The demand function is $D(p) = 0.001p(30 - 0.001p)$, for $0 < p < 30000$, where p is the price of a car. Should the dealer raise or lower the price to increase the revenue? What price will give the dealer maximum revenue?

6. Evaluate the given integral

   (a)

$$\int x(x+999)^7\, dx,$$

   (b)

$$\int \frac{\sec^2 \theta}{\tan^2 \theta - \tan \theta - 6}\, d\theta.$$

   (c)

$$\int \sec^3 t\, dt$$

   (d)

$$\int_{-1}^{1} x^{-1/3}\, dx$$

7. Let a curve Γ be defined by: $\Gamma : y = \sqrt{4 - 4x^2}$, $x \in [-1, 1]$

   (a) Set up a definite integral for the arc length of Γ.

   (b) Set up a definite integral for the surface area generated by revolving Γ about x-axis.

8. Solve the IVP, explicitly, if possible $y' = \frac{x-1}{y}, y(0) = -2$.

9. The plot here shows the relationship between the specific partial pressure of oxygen (pO2, measured in mm Hg) and the saturation level of hemoglobin (y = 1 would mean that no more oxygen can bind). Determine whether $f_1(x) = x/(27 + x)$ or $f_2(x) = x^3/(27^3 + x^3)$ is a better model for this data by finding extrema, inflection points and asymptotes (for $x > 0$) for each function and comparing to the graph.