Calculus I **Final** **Jan. 22, 2007**

Name: _____

Student ID number: _____

Guidelines for the test:

- Put your name or student ID number on every page.
- There are 14 problems: 9 problems in Part I (6 points each) and 5 problems in Part II (10 points each). If you got more than 100 points, you will only get 100 points. (超過100分以100分計算)
- The exam is closed book; calculators are not allowed.
- There is no partial credit for the Problems in the Part I (multiple-choice (選擇) and fill-in (填充) problems).
- For problems in the Part II (problem-solving (計算題) problems), please show all work, unless instructed otherwise. Partial credit will be given only for work shown. Print as legibly as possible - correct answers may have points taken off, if they're illegible.
- Mark the final answer.

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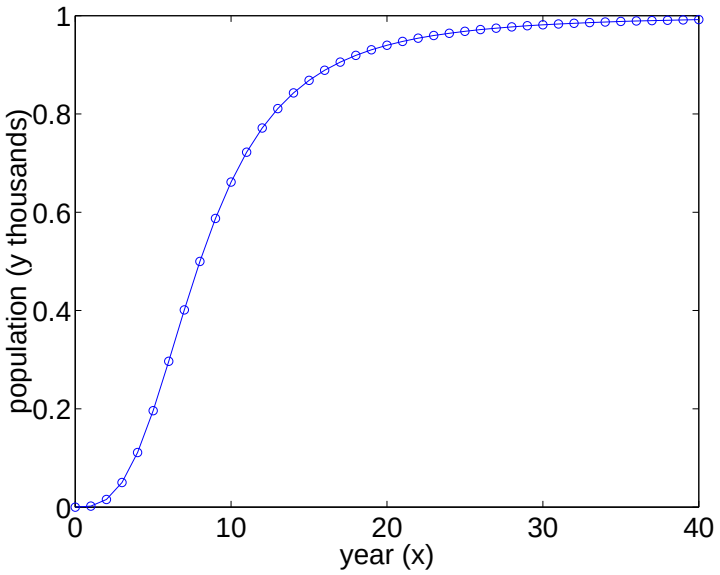
Part I: (6 points for each problem)
Multiple Choice - Single Answer (選擇題- 單選題).

(1) The plot below shows the growth of the population on an small island. Choose a better model from the options for this data? (Hint: find extrema, inflection points and asymptotes)

(A) $f(x) = \frac{x}{8+x}$ (B) $g(x) = \frac{x^3}{8^3+x^3}$



(2) An automobile dealer is selling cars at a price of \$26,000. The demand function is $D(p) = 0.001p(30 - 0.001p)$, for $0 \le p \le 30000$, where p is the price of a car. Should the dealer raise or lower the price to increase the revenue? (Revenue function: $R(p) = p \cdot D(p)$)

A) Raise the price
B) lower the price,
C) Keep the price unchanged (現在這個價錢收益最大)

(3) A Region is bounded by two curves: $y = x^3$ and $y = x$. Set up a definite integral to which gives the area of the region

A) $\int_{-1}^{1} x - x^3\,dx$
B) $\int_{-1}^{0} x - x^3\,dx + \int_{0}^{1} x^3 - x\,dx$,
C) $\int_{-1}^{0} x^3 - x\,dx + \int_{0}^{1} x - x^3\,dx$
D) $\int_{-1}^{1} x^3 - x\,dx$

(4) Use formulas for indefinite integrals to evaluate

$$\int \frac{1}{x^2-4x+5}\,dx = ?$$

A) $\ln|x^2-4x+5|+C$, B) $\tan^{-1}(x)+C$
C) $\ln|x-4|+\ln|x-1|+C$, D) $\tan^{-1}(x-2)+C$.

(5) A solid is formed by revolving the disk $(x-2)^2+y^2=4$ about the $x-$axis. Set up a definite integral which gives the volume of the solid.

A) $\int_{-2}^{2}\pi(\sqrt{4-(x-2)^2})^2\,dx$ B) $\int_{0}^{4}\pi(\sqrt{4-(x-2)^2})^2\,dx$,
C) $\int_{0}^{4}2\pi(\sqrt{4-(x-2)^2})^2\,dx$ D) $\int_{-2}^{2}2\pi(\sqrt{4-(x-2)^2})^2\,dx$

(6) Given that the lifetime of a lightbulb is exponentially distributed with pdf $f(x)=6e^{-6x}$ (with x measured in years), Find the probability that the lightbulb lasts between 1 and 2 **months**.

A) $\int_{1}^{2}6e^{-6x}\,dx$ B) $\int_{1}^{2}6xe^{-6x}\,dx$,
C) $\int_{1/12}^{2/12}6e^{-6x}\,dx$ D) $\int_{1/12}^{2/12}6xe^{-6x}\,dx$

Fill-In Problems(填充)

(7) Let $f(x)=\begin{cases} x^3, & x<2 \\ Ax-2, & x\geq 2 \end{cases}$. Find A given that f is continuous at 2.

$A=$_____

(8) $\int \sqrt{x}+\frac{1}{\sqrt{x}}\,dx=$_____ .

(9) $\frac{d}{dx}(4^{x^2}+3^x+x^2)=$_____ .

Part II: (10 points for each problem)

Problem-Solving Problems (計算題 **Show all work**)

(10) Evaluate

$$\int \ln x \, dx$$

(11) Evaluate

$$\int_{-2}^{1} x(x+2)^{99} \, dx$$

(12) Use the technique of partial fraction decomposition to evaluate

$$\int \frac{7}{x^2 + 5x - 6}\, dx$$

(13) Find the derivative of $f(x) = \ln\left(x\sqrt{x^2+1}/\sqrt{5x+2}\right)$.

(14) Given that $f(x) = x^3 - 3x$, find the local maximum and minimum values of $f(x)$.

- Rule of exponents

  For any integers m and n,

  $x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$ For any real p, $x^{-p} = \frac{1}{x^p}$

  For any real p and q, $(x^p)^q = x^{pq}$ For any real p and q, $x^p \cdot x^q = x^{p+q}$

- properties of logarithm function

  For any positive base $b \neq 1$ and positive numbers x and y, we have

  $\log_b(xy) = \log_b x + \log_b y$ $\log_b(x/y) = \log_b x - \log_b y$

  $\log_b(x^y) = y \log_b x$ $\log_b(x) = \frac{\ln x}{\ln b}$

- Derivative formulas

  $\frac{d}{dx} e^x = e^x$ $\frac{d}{dx} \ln x = \frac{1}{x}$

- Formulas for indefinite integrals:

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln(x+\sqrt{x^2+a^2}) \quad \text{or} \quad \sinh^{-1}\frac{x}{a}$$

$$\int \frac{1}{ax^2+bx+c}\,dx = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} & \text{if} \quad b^2-4ac<0 \\ \frac{1}{\sqrt{b^2-4ac}} \ln(\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}) & \text{if} \quad b^2-4ac>0 \end{cases}$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\frac{x-a}{x+a} \quad \text{or} \quad -\frac{1}{a}\coth^{-1}\frac{x}{a}$$