Name: _____

Student ID number: _____

Guidelines for the test:

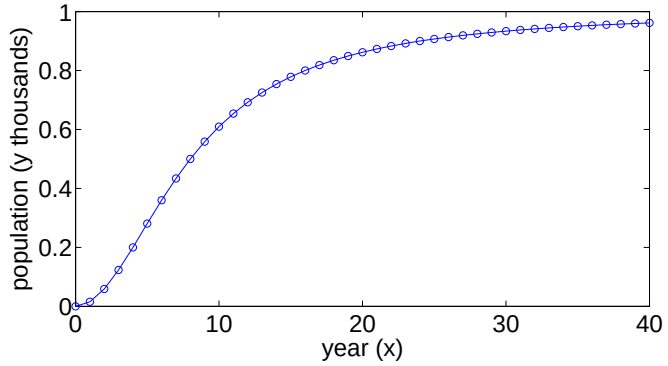
- Put your name or student ID number on every page.
- There are 13 problems: 3 problems in Part I and 10 problems in Part II .
- The exam is closed book; calculators are not allowed.
- There is no partial credit for problems in the Part I (選擇, 填充及是非).
- For problems in the Part II (problem-solving (計算與證明題) problems), please show all work, unless instructed otherwise. Partial credit will be given only for work shown. Print as legibly as possible - correct answers may have points taken off, if they're illegible.
- **Mark the final answer**.

Calculus **Student ID number:** _____

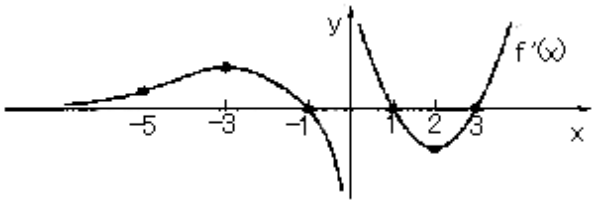
Part I: 單選, 填充, 是非題 (5 points for each problem)

1. (5 pts) The plot below shows the growth of the population on an small island. Choose a better model from the options for this data? (Hint: find critical points, inflection points and asymptotes)

   (A) $\dfrac{x}{x+8}$ (B) $\dfrac{x^2}{x^2-8^2}$ (C) $\dfrac{x}{x-8}$ (D) $\dfrac{x^2}{x^2+8^2}$



2. (5 pts) $f(x)$ is a **continuous** function on $(-\infty, \infty)$ and the graph of its **derivative**, $f'(x)$, is shown in the figure below. (Note: $\lim_{x\to-\infty} f'(x) = 0$; $\lim_{x\to\infty} f'(x) = \infty$)



   Answer the following True/False questions (True ⇒ ◯ ; False ⇒ ×).

   - _____ $f(x)$ has a horizontal asymptote.
   - _____ $f(x)$ has a vertical asymptote.
   - _____ $(1, f(1))$ is an inflection point.
   - _____ $(2, f(2))$ is an inflection point.
   - _____ f has a local minimum at $x = 0$.

3. (5 pts)A Region is bounded by two curves: $y = x^3$ and $y = x$. Set up a definite integral representing the area of the region

   A) $\int_{-1}^{1} x - x^3\,dx$ B) $\int_{-1}^{0} x - x^3\,dx + \int_{0}^{1} x^3 - x\,dx$,
   C) $\int_{-1}^{0} x^3 - x\,dx + \int_{0}^{1} x - x^3\,dx$ D) $\int_{-1}^{1} x^3 - x\,dx$

Part II: Problem-Solving Problems (計算與證明題 **Show all work**)

4. (10 pts) The sequence $\{x_n\}$ is recursively defined.

$$x_{n+1} = \frac{4x_n^2}{4 + x_n^2}$$

   (a) (2 pts) Find all equilibria (fixed points) of $\{x_n\}$.

   (b) (8 pts) Determine the stability of the equilibria (fixed points).

5. (10 pts) Given that $F(x) = \int_1^{x^2} \sqrt{1 + t^2}\, dt$, for $x \geq 0$,

   (a) $F'(x) =?$

   (b) $(F^{-1})'(0) =?$

   Note: since $\sqrt{1 + t^2} > 0$, $F(x)$ is monotone and $F^{-1}(x)$ is well-defined.

6. (5 pts) Use formulas for indefinite integrals to evaluate $\int \frac{1}{x^2 - 4x + 11}\, dx$.

7. (5 pts) $\frac{d}{dx}(x^{\cos x})$.

8. (10 pts) $f(x) = \frac{\ln x}{x}$, $x > 0$.

   (a) Find the maximum value of $f(x)$.

   (b) Prove that $\pi^e < e^\pi$

9. (5 pts) Evaluate $\int \frac{1}{x} e^{199+\ln x}\, dx$

10. (10 pts) Evaluate $\int_e^{e^2} \frac{\sqrt{\ln x}}{x}\, dx$

11. (10 pts) Evaluate $\int \ln x\, dx$

12. (10 pts) Evaluate the indefinite integrals

(a) $\int \frac{2t+1}{t^2+2t}\,dt$

(b) $\int \frac{\cos^3 x}{\sin^2 x + 2\sin x}\,dx$

13. (10 pts) Compute: (Be sure to check whether l'Hospital's rule can be applied before you use it.)

(a) $\lim_{x\to\infty} \frac{x}{e^x}$

(b) $\lim_{x\to\infty} (1+\frac{2}{x})^x$