

Name: \_\_\_\_\_

Student ID number: \_\_\_\_\_

TA/classroom: \_\_\_\_\_

**Guidelines for the test:**

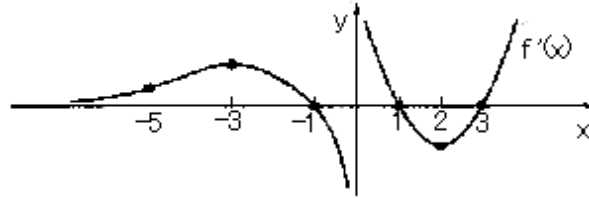
- Put your name or student ID number on every page.
- There are 11 problems
- The exam is closed book; calculators are not allowed.
- There is no partial credit for problem 1-3.
- For other problems, please show all work, unless instructed otherwise. Partial credit will be given only for work shown. Print as legibly as possible - correct answers may have points taken off, if they're illegible.
- **Mark the final answer.**

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1. (2 pts each)  $f(x)$  is a **continuous** function on  $(-\infty, \infty)$  and the graph of its derivative,  $f'(x)$ , is shown in the figure below.

(Note:  $\lim_{x \rightarrow -\infty} f'(x) = 0$ ;  $\lim_{x \rightarrow \infty} f'(x) = \infty$ )



Answer the following True/False questions (True  $\Rightarrow \bigcirc$  ; False  $\Rightarrow \times$ ).

- \_\_\_\_  $(1, f(1))$  is an inflection point.
  - \_\_\_\_  $f$  has a local maximum at  $x = -1$
  - \_\_\_\_  $f$  has a local minimum at  $x = 1$
  - \_\_\_\_  $f(x)$  has 3 critical numbers.
2. (2 pts each) Suppose  $f(x)$  is a continuous function, and  $F(x)$  is an antiderivative function of  $f(x)$ , i.e.,  $F'(x) = f(x)$ . Answer the following True/False questions (True  $\Rightarrow \bigcirc$  ; False  $\Rightarrow \times$ ).
- \_\_\_\_ If  $f(x)$  is an odd function, then  $F(x)$  is an even function.
  - \_\_\_\_ If  $f(x)$  is an even function, then  $F(x)$  is an odd function.
  - \_\_\_\_ If  $f(x)$  is a periodic function, then  $F(x)$  is a periodic function.
  - \_\_\_\_ If  $f(x)$  is monotonically increasing, then  $F(x)$  is monotonically increasing.

Note:

- The graph of an even function is symmetric with respect to the y-axis.
  - The graph of an odd function is symmetric with respect to the origin.
  - A function  $f$  is called monotonic increasing, if for all  $x$  and  $y$  such that  $x \leq y$  one has  $f(x) \leq f(y)$ .
3. (2 pts each) Answer the True/False questions (True  $\Rightarrow \bigcirc$  ; False  $\Rightarrow \times$ ).
- \_\_\_\_  $2 - \cos x$  is an antiderivative function of  $\sin x$ .
  - \_\_\_\_  $2 \sin^2 \frac{x}{2}$  is an antiderivative function of  $\sin x$ .

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4. Evaluate each of the following limits.

(a) (5 pts)  $\lim_{x \rightarrow 0^+} \sin x \ln x$

(b) (5 pts)  $\lim_{x \rightarrow 0^+} x^{\sin x}$

5. Find  $\frac{dy}{dx}$  for each of the following.

(a) (5 pts)  $y = x^{\sin x}$ ,  $x > 0$ .

(b) (5 pts)  $y = e^{2x} \frac{\sqrt{x+1}}{x^2+2} (2x+1)^5$ ,  $x > 0$ .

6. (10 pts) Given that  $F(x) = \int_1^{x^2} e^{t^2} dt$ , for  $x \geq 0$ ,

(a) Find  $F'(x)$

(b) Find  $(F^{-1})'(0)$

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7. Evaluate the given integral

(a) (5 pts)  $\int e^{2x} \sin x \, dx.$

(b) (5 pts)  $\int \frac{\sqrt{\ln x}}{x} \, dx,$

(c) (5 pts)  $\int \frac{3x}{(x+1)(x-4)} \, dx,$

8. (10 pts) Evaluate the definite integrals  $\int_1^4 e^{\sqrt{x}} \, dx$

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9. (a) (5 pts) Evaluate  $\int \cos^2 \theta d\theta$ .

(b) (5 pts) Use the trigonometric substitution to evaluate  $\int_0^1 \sqrt{1-x^2} dx$ ,

10. (5 pts) Use formulas for indefinite integrals to evaluate  $\int \frac{1}{x^2 - 4x + 5} dx$ .

11. Evaluate the given integral

(a) (5 pts)  $\int_{-1}^1 x^{-2} dx$

(b) (5 pts)  $\int_{-\infty}^{\infty} x dx$