Calculus **Midterm** **Version 1** **Nov. 15, 2006**

Name: ________________________________

Student ID number: ________________________________

Guidelines for the test:

- Put your name or student ID number on every page.
- There are 14 problems: 10 problems in Part I and 4 problems in Part II.
- The exam is closed book; calculators are not allowed.
- There is no partial credit for the Problems in the Part I (multiple-choice (選擇) and fill-in (填充) problems).
- For problems in the Part II (calculation (計算題) problems), please show all work, unless instructed otherwise. Partial credit will be given only for work shown. Print as legibly as possible - correct answers may have points taken off, if they're illegible.
- Mark the final answer.

Part I: (6 points for each problem)
Multiple Choice (**Single Choice**)

(1) Which of the following pairs of functions are inverse functions of each other on the implied domains?

A) $f(x) = |x|$; $g(x) = |x|$

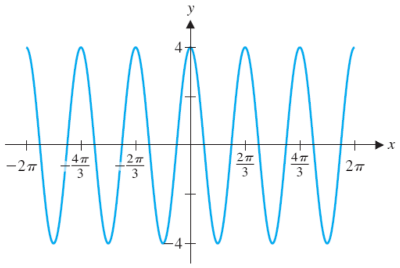
B) $f(x) = 2x - 1$, $g(x) = \frac{1}{2}x + 1$

c) $f(x) = \frac{1}{x}$; $g(x) = \frac{x}{1}$,

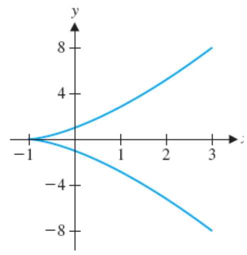
D) $f(x) = \sqrt[3]{x}$; $g(x) = x^3$.

(2) Which of the following curves is **NOT** the graph of a function?

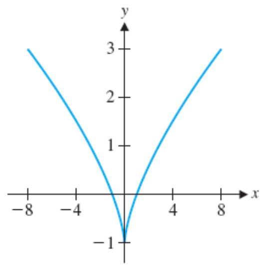
(A)



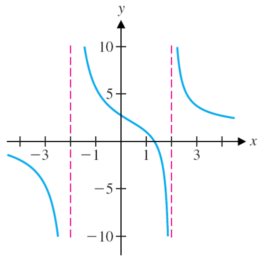
(B)



(C)

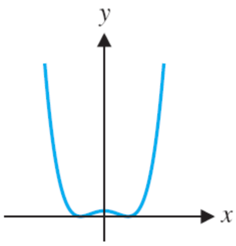


(D)

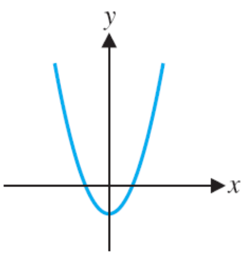


(3) Find the graph corresponding to the derivative of the given function?

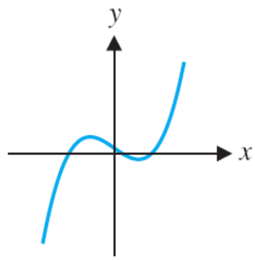
f(x)



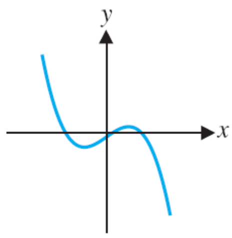
(A)



(B)



(C)



(4) Given that $f(x) = \frac{x^3}{x^2-1}$, $f'(x) = \frac{x^2(x^2-3)}{(x^2-1)^2}$, $f(0) = 0$, $f(\sqrt{3}) = \frac{3\sqrt{3}}{2}$ and $f(-\sqrt{3}) = -\frac{3\sqrt{3}}{2}$, which one of the following is **NOT** true?

A) Domain of f is $\{x \neq \pm 1\}$

B) The absolute maximum of f occurs at $x = \sqrt{3}$,

C) f has no absolute extremum,

D) f does not have a local extremum at $x = 0$

(5) $\frac{d}{dx}(x^x) = ?$

A) x^x B) $x^x(\ln x + 1)$, C) $x^x \ln x$, D) x^{x-1}

Fill-In Problems

(6) Let $f(x) = \begin{cases} 2x-3, & x<2 \\ 2, & x=2 \\ x^2-3x, & x>2 \end{cases}$.

$\lim_{x \to 2^-} f(x) + f(2) + 3 \lim_{x \to 2^+} f(x) =$ ______________________________ .

(7) Let $f(x) = \begin{cases} x^3, & x<2 \\ Ax-2, & x \geq 2 \end{cases}$. Find A given that f is continuous at 2.

$A =$ ______________________________

(8) $\lim_{x \to \infty} \dfrac{x^2}{x^2-1} =$ ______________________________

(9) $\lim_{x \to 1^-} \dfrac{x}{x^2-1} =$ ______________________________

(10) $\dfrac{d}{dx}(2e^{x^3}) =$ ______________________________ .

Part II: (10 points for each problem)

Calculation Problems (**Show all work**)

(11) Compute $f'(x)$ by definition ($f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$).

$$f(x) = \sqrt{3+x}$$

(12) If $x^2 + y^2 = 4$, use implicit differentiation to obtain $\dfrac{dy}{dx}$ in term of x and y. Find the equation of the tangent line at the point $(\sqrt{2}, \sqrt{2})$.

(13) Find $\dfrac{d}{dx}\left(\dfrac{\sqrt{x^2+4}}{x+1}\right)$

(14) Given that $f(x) = x^3 - x$, find the critical number of $f(x)$. Find the absolute maximum and absolute minimum values of the function $f(x)$ on the interval $[0, 2]$.

- Double-Angle
  $\sin 2\theta = 2\sin\theta\cos\theta$
  $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

- Rule of exponents
  For any integers m and n,
  $x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$
  For any real p, $x^{-p} = \frac{1}{x^p}$
  For any real p and q, $(x^p)^q = x^{pq}$
  For any real p and q, $x^p \cdot x^q = x^{p+q}$

- properties of logarithm function
  For any positive base $b \neq 1$ and positive numbers x and y, we have
  $\log_b(xy) = \log_b x + \log_b y$
  $\log_b(x/y) = \log_b x - \log_b y$
  $\log_b(x^y) = y\log_b x$
  $\log_b(x) = \frac{\ln x}{\ln b}$

- Derivative formulas
  $\frac{d}{dx}\sin x = \cos x,$
  $\frac{d}{dx}\cos x = -\sin x,$
  $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$
  $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$,for $-1 < x < 1$
  $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2},$
  $\frac{d}{dx}\cot^{-1} x = -\frac{1}{1-x^2},$
  $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$,for $|x| > 1$
  $\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$for $|x| > 1$
  $\frac{d}{dx}e^x = e^x$
  $\frac{d}{dx}\ln x = \frac{1}{x}$