Calculus 11/14/2005

Name:______________________________ ID:______________________________

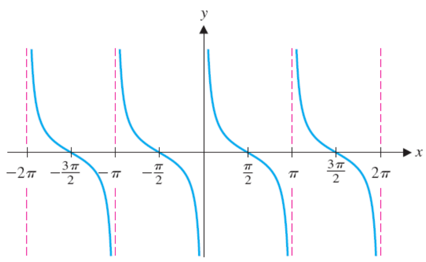
1. Which of the following pairs of functions are inverse functions of each other on the implied domains? (may have more than one answer)

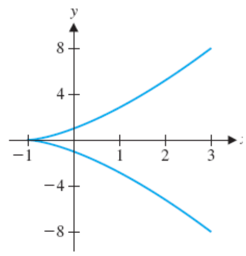
   A) $f(x) = |x|$; $g(x) = |x|$ B) $f(x) = \frac{1}{x}$; $g(x) = \frac{x}{1}$,

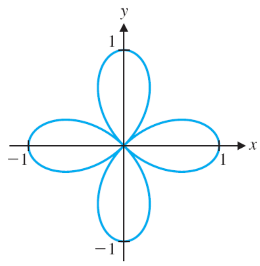
   C) $f(x) = \frac{1}{x}$; $g(x) = \frac{1}{x}$, D) $f(x) = \sqrt{x}$; $g(x) = x^2$, for $x \geq 0$.

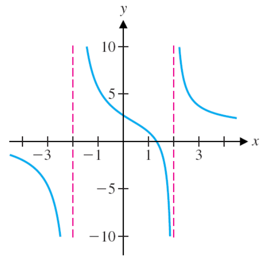
2. Which of the following curves is **NOT** the graph of a function?(may have more than one answer)

   (A) (B) (C) (D)









   A) graph A, B) graph B, C) graph C D) graph D

3. Find $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$.

4. Let $f(x) = \begin{cases} x^2, & x < 1 \\ Ax - 2, & x \geq 1 \end{cases}$. Find A given that f is continuout at 1.

5. Find $\lim_{x \to 0} \dfrac{\tan 3x}{2x^2 + 5x}$.

6. Find all discontinuities of $f(x)$. For each discontinuity that is removable, define a new function that removes the discontinuity.

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

7. Find the rate of change of $y = 1/[x(x+1)]$ with respect to x at $x = 2$.

   Ans:: $-\frac{5}{36}$

8. Find dy/dx at $x = 2$ if $y = (s+3)^2$, $s = \sqrt{t-3}$, $t = x^2$.

   Ans:: 16. Hint:$\frac{dy}{dx} = \frac{dy}{ds}\frac{ds}{dt}\frac{dt}{dx}$

9. If $g(x) = f(x^2 + 1)$, find $g'(1)$ given that $f'(2) = 3$.

   Ans:: 6. Hint: $g'(x) = f'(x^2 + 1) \cdot (2x)$

10. Let $f(x) = \begin{cases} 2x - 1, & x \leq 2 \\ x^2 - x, & x > 2 \end{cases}$. Find $\lim_{x \to 2^-} f(x) + f(2) + 3 \lim_{x \to 2^+} f(x)$.

11. Using the definition of detivative (limits), compute $f'(x)$.

$$f(x) = \sqrt{x+2}$$

12. Find $\dfrac{d^2}{dx^2}\left(x^2 \sin 6x\right)$

13. If $x^2 + y^2 = 4$, use implicit differentiation to obtain $\dfrac{dy}{dx}$ in term of x and y.

14. Find the equation of the tangent line to the curve $x^2 + xy + 2y^2 = 28$ at the point $(-2, -3)$.

15. Find $\dfrac{d}{dx}\left(\dfrac{\sqrt{x^2+1}}{x+2}\right)$, $\dfrac{d}{dx}(3e^{x^2})$, $\dfrac{d}{dx}(x^{x^2})$, $\dfrac{d}{dx}(\sin^{-1} x^2)$

16. A particle is moving along the parabola $y^2 = 4(x+2)$. As it passes through the point $(7, 6)$, its y–coordinate is increasing at the rate of 3 units per second. How fast is the x–coordinate changing at this instance?

17. Find the absolute maximum and absolute minimum values of the function $f(x) = \frac{x}{x^3+2}$ on the interval $[0, 2]$.

18. Given $f(x) = \frac{x}{x^2-1}$, find:

    - Domain of the function;
    - Horizontal and Vertical Asymptotes;
    - Interval of increasing and decreasing;
    - Critical points and local extrema;
    - Determine where the graph is concave up and concave down and locate any inflection points;
    - Locate x- and y- intercepts, if any;

    and draw a graph of the function showing all significant features.

- Double-Angle

  $\sin 2\theta = 2 \sin\theta \cos\theta$

  $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

- Derivative formulas

  $\frac{d}{dx}\sin x = \cos x$,

  $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$

  $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$,

  $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$,for $|x| > 1$

  $\frac{d}{dx}e^x = e^x$

  $\frac{d}{dx}\cos x = -\sin x$,

  $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$,for $-1 < x < 1$

  $\frac{d}{dx}\cot^{-1} x = -\frac{1}{1-x^2}$,

  $\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$for $|x| > 1$