Calculus 11/14/2005

Name:____________________________ ID:____________________________

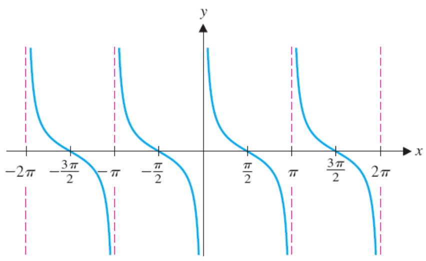
1. Which of the following pairs of functions are inverse functions of each other on the implied domains? (may have more than one answer)

   A) $f(x) = |x|;\ g(x) = |x|$ B) $f(x) = \frac{1}{x};\ g(x) = \frac{x}{1},$
   C) $f(x) = \frac{1}{x};\ g(x) = \frac{1}{x},$ D) $f(x) = \sqrt{x};\ g(x) = x^2,$ for $x \geq 0.$

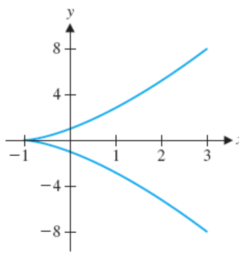
   Ans::C,D

2. Which of the following curves is **NOT** the graph of a function?(may have more than one answer)

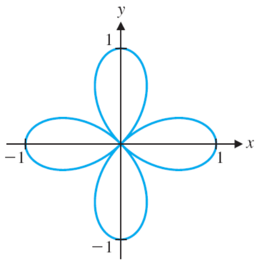
   (A)



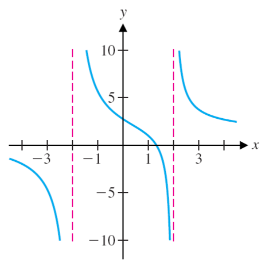
   (B)



   (C)



   (D)



   Ans:: B,C

3. Find $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$.

   Ans:: $\frac{1}{2}$

4. Let $f(x) = \begin{cases} x^2, & x < 1 \\ Ax - 2, & x \geq 1 \end{cases}$. Find A given that f is continuout at 1.

   Ans:: $A = 3$

5. Find $\lim_{x \to 0} \dfrac{\tan 3x}{2x^2 + 5x}$.

   Ans:: $\frac{3}{5}$

6. Find all discontinuities of $f(x)$. For each discontinuity that is removable, define a new function that removes the discontinuity.

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

   Ans:: $f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

7. Find the rate of change of $y = 1/[x(x+1)]$ with respect to x at $x = 2$.

   Ans:: $-\frac{5}{36}$

8. Find dy/dx at $x = 2$ if $y = (s+3)^2$, $s = \sqrt{t-3}$, $t = x^2$.

   Ans:: 16. Hint:$\frac{dy}{dx} = \frac{dy}{ds}\frac{ds}{dt}\frac{dt}{dx}$

9. If $g(x) = f(x^2+1)$, find $g'(1)$ given that $f'(2) = 3$.

   Ans:: 6. Hint: $g'(x) = f'(x^2+1) \cdot (2x)$

10. Let $f(x) = \begin{cases} 2x-1, & x \leq 2 \\ x^2 - x, & x > 2 \end{cases}$. Find $\lim_{x\to 2^-} f(x) + f(2) + 3\lim_{x\to 2^+} f(x)$.

    Ans:: 12

11. Using the definition of detivative (limits), compute $f'(x)$.

$$f(x) = \sqrt{x+2}$$

    Ans:: $\frac{1}{2\sqrt{x+2}}$

12. Find $\frac{d^2}{dx^2}\left(x^2 \sin 6x\right)$

    Ans:: $2\sin 6x + 24x\cos 6x - 36x^2\sin 6x$

13. If $x^2 + y^2 = 4$, use implicit differentiation to obtain $\frac{dy}{dx}$ in term of x and y.

    Ans:: $-\frac{x}{y}$

14. Find the equation of the tangent line to the curve $x^2+xy+2y^2 = 28$ at the point $(-2, -3)$.

    Ans:: $m = -\frac{1}{2}$; $(y+3) = -\frac{1}{2}(x+2)$

15. Find $\frac{d}{dx}\left(\frac{\sqrt{x^2+1}}{x+2}\right)$, $\frac{d}{dx}(3e^{x^2})$, $\frac{d}{dx}(x^{x^2})$, $\frac{d}{dx}(\sin^{-1} x^2)$

    Ans:: $\frac{2x-1}{(x+2)^2\sqrt{x^2+1}}$; $6xe^{x^2}$; $x^{x^2}(2x\ln x + x)$; $\frac{2x}{\sqrt{1-x^4}}$

16. A particle is moving along the parabola $y^2 = 4(x+2)$. As it passes through the point $(7, 6)$, its y–coordinate is increasing at the rate of 3 units per second. How fast is the x–coordinate changing at this instance?

    Ans:: $2y\frac{dy}{dt} = 4\frac{dx}{dt}$, $\frac{dx}{dt} = 9$

17. Find the absolute maximum and absolute minimum values of the function $f(x) = \frac{x}{x^3+2}$ on the interval $[0, 2]$.

    Ans:: abs max:$f(1) = \frac{1}{3}$; abs min: $f(0) = 0$

18. Given $f(x) = \frac{x}{x^2-1}$, find:

    - Domain of the function;

      Ans::$x \neq 1$

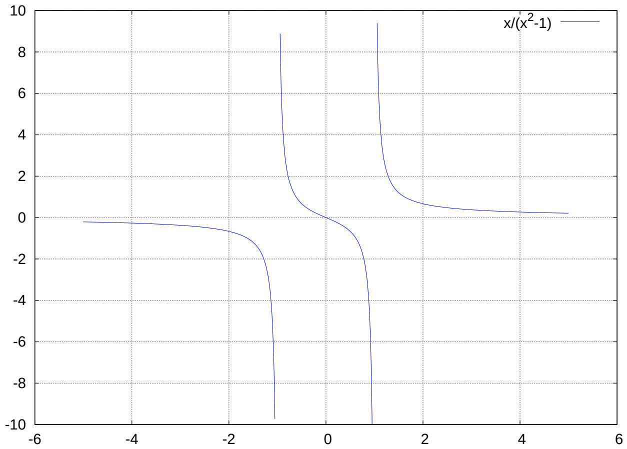
    - Horizontal and Vertical Asymptotes;

      Ans:: H:$y = 0$; V:$x = 1$, $x = -1$.
      ($\lim_{x\to -1^-} f(x) = -\infty$, $\lim_{x\to -1^+} f(x) = \infty$,$\lim_{x\to 1^-} f(x) = -\infty$,$\lim_{x\to 1^+} f(x) = \infty$)

- Interval of increasing and decreasing;

  Ans:: Interval of increasing: None;
  Interval of decreasing: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
- Critical points and local extrema; **Ans:**: None
- Locate x- and y- intercepts, if any; **Ans:**: $(0, 0)$

and draw a graph of the function showing all significant features.



- Double-Angle

  $\sin 2\theta = 2\sin\theta\cos\theta$ $\quad\quad$ $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

- Derivative formulas

  $\frac{d}{dx}\sin x = \cos x,$ $\quad\quad$ $\frac{d}{dx}\cos x = -\sin x,$
  $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$ $\quad\quad$ $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$,for $-1 < x < 1$
  $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2},$ $\quad\quad$ $\frac{d}{dx}\cot^{-1} x = -\frac{1}{1-x^2},$
  $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$,for $|x| > 1$ $\quad\quad$ $\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$for $|x| > 1$
  $\frac{d}{dx}e^x = e^x$