Calculus I, 2006 Practice problems

Final Exam:

Time: 1/22(M), 3:10-5:00; Place: 格致堂

Chap 1: Sec. 1.2-Sec. 1.5:

1. Let $f(x) = \begin{cases} |x+2| & \text{for } x \le 0; \\ 2+x^2 & \text{for } 0 < x < 2; \\ x^3 & \text{for } x \ge 2 \end{cases}$. Find (a) $\lim_{x\to 0^-} f(x)$, (b)$\lim_{x\to 0^+} f(x)$, (c) $\lim_{x\to 2^-} f(x)$, (d)$\lim_{x\to 2^+} f(x)$, (e) $\lim_{x\to 0} f(x)$, (f) $\lim_{x\to 2} f(x)$.
2. Let $f(x) = \begin{cases} cx-2 & \text{for } x \le 2; \\ cx^2+2 & \text{for } x > 2 \end{cases}$ Find c such that $f(x)$ is continuous.
3. Determine the intervals on which $f(x) = \ln(1-x^2)$ is continuous.
4. Compute (i) $\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x}$ (ii) $\lim_{x\to 1^-} \frac{2x}{x^2-1}$

Chap 2: Sec. 2.3-Sec. 2.9:

1. Find the tangent line to the curve $y = x^3 - 4x^2 + 2x + 1$ at the point $(1, 0)$.
2. Let $y = e^{x^2} \cdot (x^2+x+1) \cdot \sqrt{3x+1}/(x^2-1)$. Find $\frac{dy}{dx}$.
3. The equation $7x^2y^3 - 5xy^2 - 4y = 7$ defines y implicitly as a function of x. Find $\frac{dy}{dx}$.
4. Find the detivative of (i) $f(x) = x^{2x}$; (ii) $g(x) = \frac{x^2-x}{3x+1}$; (iii) $h(x) = \ln\sqrt{\frac{3x+1}{5x+2}}$
5. Determine if $f(x) = x^7 + 2x^3 - 2006$ is increasing, decreasing or neither. Prove $f(x) = 0$ has exactly one solution.

Chap 3: Sec. 3.1-Sec. 3.8:

1. Estimate $\sqrt[3]{8.02}$ by the method of linear approximation (i.e., by differentials).
2. Find the asymptotes of
   (i) $f(x) = \frac{(3x-1)^2}{9x^2-4}$. (ii) $f(x) = \frac{(3x-1)^2}{9x^2-1}$.(iii)$f(x) = \frac{(3x-1)^2}{x-1}$
3. Let $f(x) = 2x^3 - 3x^2 - 12x$. Find the relative extrema of $f(x)$.
4. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 9x^2 + 12x$ over the interval $[0, 2]$.
5. Determine the concavity of $f(x) = 4x^3 - x^4$.
6. If 300 cm^2 of material is available to make a box with square base and an open top, find the largest possible volume of the box. Explain why your answer is the absolute maximum.
7. Sketch the graph of the continuous function f that satisfies the conditions:

$$\begin{aligned} f''(x) &> 0 \quad \text{if } |x| > 2, \quad f''(x) < 0 \quad \text{if } |x| < 2; \\ f'(0) &= 0, \quad f'(x) > 0, \quad \text{if } x < 0, \quad f'(x) < 0, \quad \text{if } x > 0; \\ f(0) &= 1, \quad f(2) = \frac{1}{2}, \quad f(x) > 0 \quad \text{for all } x, \text{and } f \text{ is and even function.} \end{aligned}$$

8. An automobile dealer is selling cars at a price of \$12,000. The demand function is $D(p) = 2(15 - 0.001p)^2$, where p is the price of a car. Should the dealer raise or lower the price to increase the revenue?

9. Compute:
(i) $\lim_{x\to 0}(\frac{1}{\ln(x+1)} - \frac{1}{x})$; (ii) $\lim_{x\to 1^+} \frac{\ln x}{(x-1)^2}$

Chap 4: Sec. 4.2-Sec. 4.8 (Integration Tables),

1. Let $f(x) = x + 1$
(a) Divide the interval $[0, 5]$ into n equal parts, and using right endpoints find an expression for the Riemann sum R_n.
(b) Using the answer you got from part(a), calculate $\lim_{n\to\infty} R_n$ (without using antiderivatives).

2. Evaluate the given integral
(i) $\int x(x+1)^9\,dx$, (ii) $\int \frac{dx}{e^x\sqrt{4+e^{2x}}}$. (iii) $\int \frac{\ln x}{x\sqrt{1+\ln x}}\,dx$, (iv) $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$.

3. Evaluate the given integral
(i) $\int \sqrt{x}e^{\sqrt{x}}\,dx =?$; (ii) $\int \frac{\sqrt{\ln x}}{x}\,dx =?$; (iii) $\int \frac{x}{x+4}\,dx =?$; (iv) $\int (\ln x)^2\,dx =?$;

4. Evaluate the given integral
(i) $\int \frac{\ln x}{x}\,dx =?$; (ii) $\int \ln(x^2)\,dx =?$; (iii) $\int \frac{3x}{x^2-3x-4}\,dx =?$; (iv) $\int \frac{-2x^2+4}{x^3+2x^2+x}\,dx =?$

5. Evaluate the definite integrals:
(i) $\int_1^4 \sqrt{x}e^{\sqrt{x}}\,dx =?$; (ii) $\int_1^e (\ln x)^2\,dx =?$;

Chap 5: Sec. 5.1, Sec. 5.2 (Volumes by slicing and the method of disks) and Sec. 5.6.

1. Find the region bounded by the parabola $x = 2 - y^2$ and the line $y = x$.

2. A solid is formed by revolving the circular disk $(x-5)^2 + y^2 = 4$ about the y-axis. Set up, **but do not evaluate**, a definite integral which give the volume of the solid.

3. Given that the lifetime of a lightbulb is exponentially distributed with pdf $f(x) = 6e^{-6x}$ (with x measured in years), Find the probability that the lightbulb lasts between 1 and 2 months.