

Final Exam:

Time: 1/22(M), 3:10-5:00; Place: 格致堂

Chap 1: Sec. 1.2-Sec. 1.5:

1. Let $f(x) = \begin{cases} |x+2| & \text{for } x \leq 0; \\ 2+x^2 & \text{for } 0 < x < 2; \\ x^3 & \text{for } x \geq 2 \end{cases}$. Find (a) $\lim_{x \rightarrow 0^-} f(x)$, (b) $\lim_{x \rightarrow 0^+} f(x)$, (c) $\lim_{x \rightarrow 2^-} f(x)$, (d) $\lim_{x \rightarrow 2^+} f(x)$, (e) $\lim_{x \rightarrow 0} f(x)$, (f) $\lim_{x \rightarrow 2} f(x)$.
 Ans: (a) 2 (b) 2 (c) 6 (d) 8 (e) 2 (f) DNE

2. Let $f(x) = \begin{cases} cx - 2 & \text{for } x \leq 2; \\ cx^2 + 2 & \text{for } x > 2 \end{cases}$ Find c such that $f(x)$ is continuous.
 Ans: $c = -2$

3. Determine the intervals on which $f(x) = \ln(1 - x^2)$ is continuous.
 Ans: $1 - x^2 > 0$ or $(-1, 1)$

4. Compute (i) $\lim_{x \rightarrow 0} \frac{\sqrt{x+9}-3}{x}$ (ii) $\lim_{x \rightarrow 1^-} \frac{2x}{x^2-1}$
 Ans: (i) 6 (ii) $-\infty$

Chap 2: Sec. 2.3-Sec. 2.9:

1. Find the tangent line to the curve $y = x^3 - 4x^2 + 2x + 1$ at the point $(1, 0)$.
 Ans: $\frac{dy}{dx} = 3x^2 - 8x + 2$; slope = -5 ; $y - 0 = -5(x - 1)$
2. Let $y = e^{x^2} \cdot (x^2 + x + 1) \cdot \sqrt{3x+1}/(x^2 - 1)$. Find $\frac{dy}{dx}$.
 Hint: Take ln on both side.
3. The equation $7x^2y^3 - 5xy^2 - 4y = 7$ defines y implicitly as a function of x . Find $\frac{dy}{dx}$.
 Ans: $\frac{14xy^3 - 5y^2}{4 + 10xy - 21x^2y^2}$
4. Find the derivative of (i) $f(x) = x^{2x}$; (ii) $g(x) = \frac{x^2-x}{3x+1}$; (iii) $h(x) = \ln \sqrt{\frac{3x+1}{5x+2}}$ (assuming $3x+1 > 0$)
 Ans: (i) $2(\ln x + 1)x^{2x}$; (ii) $\frac{(2x-1)(3x+1) - (x^2-x)(3)}{(3x+1)^2} = \frac{(3x-1)(x+1)}{(3x+1)^2}$; (iii) $= \frac{1}{2} \left(\frac{3}{3x+1} - \frac{5}{5x+2} \right)$
5. Determine if $f(x) = x^7 + 2x^3 - 2006$ is increasing, decreasing or neither. Prove $f(x) = 0$ has exactly one solution.
 Hint: Sec. 2.9 example 9.1

Chap 3: Sec. 3.1-Sec. 3.8:

1. Estimate $\sqrt[3]{8.02}$ by the method of linear approximation (i.e., by differentials).
 Ans: $f(x) = x^{1/3}$; $f(x) \approx f(8) + f'(8)(x - 8)$; $\sqrt[3]{8.02} \approx 2 + \frac{1}{12}(0.02) \approx 2.001667$
2. Find the asymptotes of
 (i) $f(x) = \frac{(3x-1)^2}{9x^2-4}$. Ans: V: $x = 2/3, x = -2/3$; H: $y = 1$
 (ii) $f(x) = \frac{(3x-1)^2}{9x^2-1}$. Ans: V: $x = -1/3$; H: $y = 1$
 (iii) $f(x) = \frac{(3x-1)^2}{x-1}$ Ans: V: $x = 1$; H: none; S: $y = 9x + 3$.
3. Let $f(x) = 2x^3 - 3x^2 - 12x$. Find the relative extrema of $f(x)$.
 Ans: local max at $x = -1$; local min at $x = 2$; no abs. max/min

4. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 9x^2 + 12x$ over the interval $[0, 2]$.

Ans: Abs max: $f(1) = 5$; abs min: $f(0) = 0$

5. Determine the concavity of $f(x) = 4x^3 - x^4$.

Ans: Concave up: $(-\infty, 0) \cup (2, \infty)$; Concave down: $(0, 2)$

6. If 300 cm^2 of material is available to make a box with square base and an open top, find the largest possible volume of the box. Explain why your answer is the absolute maximum. Hint: example 6.2

7. Sketch the graph of the continuous function f that satisfies the conditions:

$$\begin{aligned} f''(x) &> 0 \quad \text{if } |x| > 2, & f''(x) < 0 & \quad \text{if } |x| < 2; \\ f'(0) &= 0, & f'(x) > 0, & \quad \text{if } x < 0, & f'(x) < 0, & \quad \text{if } x > 0; \\ f(0) &= 1, & f(2) = \frac{1}{2}, & & f(x) > 0 & \quad \text{for all } x, \text{ and } f \text{ is an even function.} \end{aligned}$$

Ans: The graph looks like $e^{-x^2/8}$

8. An automobile dealer is selling cars at a price of \$12,000. The demand function is $D(p) = 2(15 - 0.001p)^2$, where p is the price of a car. Should the dealer raise or lower the price to increase the revenue? Hint: Find the derivative of the revenue function, $R(p) = p \cdot D(p)$

9. Compute:

(i) $\lim_{x \rightarrow 0} (\frac{1}{\ln(x+1)} - \frac{1}{x})$; (ii) $\lim_{x \rightarrow 1^+} \frac{\ln x}{(x-1)^2}$

Ans (i) Sec 3.2 example 2.7 (ii) Sec 3.2 exercise 38

Chap 4: Sec. 4.2-Sec. 4.8 (Integration Tables),

1. Let $f(x) = x + 1$

(a) Divide the interval $[0, 5]$ into n equal parts, and using right endpoints find an expression for the Riemann sum R_n .

Hint: $1 + 2 + \dots + n = \frac{n(n+1)}{2}$

(b) Using the answer you got from part(a), calculate $\lim_{n \rightarrow \infty} R_n$ (without using antiderivatives).

2. Evaluate the given integral

(i) $\int x(x+1)^9 dx$, Hint: $u = x + 1$

(ii) $\int \frac{dx}{e^x \sqrt{4+e^{2x}}}$. Hint: $u = e^x$, 积分表

(iii) $\int \frac{\ln x}{x\sqrt{1+\ln x}} dx$, Hint: $u = \ln x$

(iv) $\int \frac{x^3}{\sqrt{x^2+1}} dx$. Hint: $u = 1 + x^2$

3. Evaluate the given integral

(i) $\int \sqrt{x} e^{\sqrt{x}} dx = ?$; (ii) $\int \frac{\sqrt{\ln x}}{x} dx = ?$; (iii) $\int \frac{x}{x+4} dx = ?$; (iv) $\int (\ln x)^2 dx = ?$;

Hint: (i) $u = \sqrt{x}$; (ii) $u = \ln x$; (iii) $= x - 4 \ln|x+4|$; (iv) $u = (\ln x)^2$; $dv = dx$

4. Evaluate the given integral

(i) $\int \frac{\ln x}{x} dx = ?$; (ii) $\int \ln(x^2) dx = ?$; (iii) $\int \frac{3x}{x^2-3x-4} dx = ?$; (iv) $\int \frac{-2x^2+4}{x^3+2x^2+x} dx = ?$

Hint: (i) $u = \ln x$; (ii) $u = \ln x^2$; $dv = dx$; (iii) $x^2 - 3x - 4 = (x-4)(x+1)$;

(iv) $x^3 + 2x^2 + x = x(x+1)^2$

5. Evaluate the definite integrals:

(i) $\int_1^4 \sqrt{x} e^{\sqrt{x}} dx = ?$; (ii) $\int_1^e (\ln x)^2 dx = ?$;

Chap 5: Sec. 5.1, Sec. 5.2 (Volumes by slicing and the method of disks) and Sec. 5.6.

1. Find the region bounded by the parabola $x = 2 - y^2$ and the line $y = x$.

Ans: $\int_{-2}^1 (2 - y^2) - y \, dy = \dots$

2. A solid is formed by revolving the circular disk $(x - 5)^2 + y^2 = 4$ about the y -axis. Set up, **but do not evaluate**, a definite integral which give the volume of the solid.

Ans: $\int_{-2}^2 \pi[(5 + \sqrt{4 - y^2})^2 - (5 - \sqrt{4 - y^2})^2] \, dy$

3. Given that the lifetime of a lightbulb is exponentially distributed with pdf $f(x) = 6e^{-6x}$ (with x measured in years), Find the probability that the lightbulb lasts between 1 and 2 months.

Ans: $\int_{1/12}^{1/6} 6e^{-6x} \, dx = \dots$