

Quiz 6
Nov. 8, 2006

1. (6 pts) Compute the derivative function $f'(x)$ by definition ($f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$).

$$\begin{aligned} f(x) &= \sqrt{x+2} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}} \\ &= \lim_{h \rightarrow 0} \frac{(x+h+2) - (x+2)}{h(\sqrt{x+h+2} + \sqrt{x+2})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+2} + \sqrt{x+2}} \\ &= \frac{1}{2\sqrt{x+2}} \end{aligned}$$

2. (7 pts) Find the derivative of $f(x) = \frac{x^2}{\sqrt{x^2+1}}$.

$$\begin{aligned} & \left(\frac{x^2}{\sqrt{x^2+1}} \right)' \\ &= \frac{(x^2)' \sqrt{x^2+1} - x^2 (\sqrt{x^2+1})'}{(\sqrt{x^2+1})^2} \\ &= \frac{2x \sqrt{x^2+1} - x^2 (x^2+1)^{-1/2} \frac{1}{2} (2x)}{(\sqrt{x^2+1})^2} \\ &= \frac{2x^3 + 2x - x^3}{(\sqrt{x^2+1})^3} \\ &= \frac{x^3 + 2x}{(\sqrt{x^2+1})^3} \end{aligned}$$

3. (7 pts) Find the derivative of $f(x) = x^2 \sqrt{x^2+1}$.

$$\begin{aligned} & (x^2 \sqrt{x^2+1})' \\ &= (x^2)' \sqrt{x^2+1} + x^2 (\sqrt{x^2+1})' \\ &= 2x \sqrt{x^2+1} + x^2 (x^2+1)^{-1/2} \frac{1}{2} (2x) \\ &= \frac{2x^3 + 2x + x^3}{\sqrt{x^2+1}} \\ &= \frac{3x^3 + 2x}{\sqrt{x^2+1}} \end{aligned}$$