Steps to sketch the curve of a given function (and related problems):

- Example 1 $f(x) = 3x^4 + 4x^3 - 6x^2 - 12x + 12$:

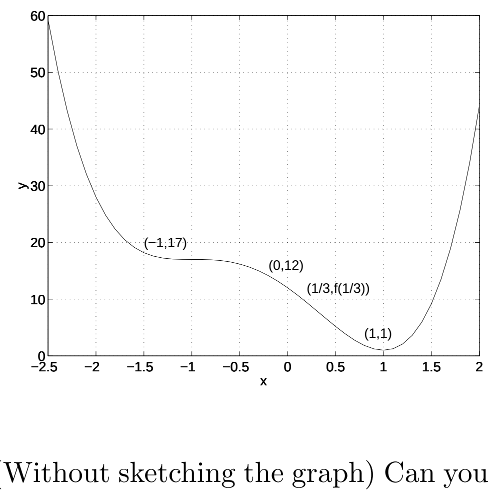
  1. Domain of the function $= (-\infty.\infty)$
  2. Horizontal and Vertical Asymptotes: None (f(x) is a polynomial).
  3. $f'(x) = 12x^3 + 12x^2 - 12x - 12 = 12(x+1)^2(x-1)$;
     Critical numbers: $x = 1$ and $x = -1$.
  4. Interval of increasing and decreasing: Note: first divide the domain into subintervals according to the location of critical numbers.

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
x	-2	0	2
$f'(x)$	-	-	+
$f(x)$	Dec	Dec	Inc

     Interval of increasing: $(1, \infty)$; Interval of decreasing: $(-\infty, 1)$
  5. Local maximum: None; local minimum at $x = 1$ (1st derivative test)
  6. $f''(x) = 36x^2 + 24x - 12 = 12(x+1)(3x-1)$;
  7. Determine where the graph is concave up and concave down: Note: first divide the domain into subintervals according to the location where $f''(x) = 0$.

	$(-\infty, -1)$	$(-1, 1/3)$	$(1/3, \infty)$
x	-2	0	1
$f''(x)$	+	-	+
Concavity of $f(x)$	Up	Down	Up

     Concave Up: $(-\infty, -1) \cup (1, \infty)$; Concave Down: $(-1, 1/3)$
  8. Inflection points: $x = -1$ and $x = 1/3$. (by table above)
  9. Locate x- and y- intercepts: No x-intercept; y-intercept: $(0, 12)$
  10. graph of the function showing all significant features.



  11. Other questions: (Without sketching the graph) Can you find the abs max/min of $f(x)$ on $[-2, 2]$, $[-2, 0]$ and $(-\infty, \infty)$? How do you know that $f(x)$ has no x-intercept (or $f(x) \neq 0$)?

- Given $f(x) = \frac{x^2}{x^2-4}$:

1. Domain of the function $= \{x \in \mathbb{R} | x \neq \pm 2\}$
2. Horizontal and Vertical Asymptotes:
   H-asymptote: $\lim_{x\to\pm\infty} \frac{x^2}{x^2-4} = \lim_{x\to\pm\infty} \frac{1}{1-4/x^2} = 1 \quad \Rightarrow \quad y = 1.$
   V-asymptotes: $x = -2$ and $x = 2$

$$\lim_{x\to-2^-} \frac{x^2}{x^2-4} = \infty \quad \lim_{x\to-2^+} \frac{x^2}{x^2-4} = -\infty$$
$$\lim_{x\to2^-} \frac{x^2}{x^2-4} = -\infty \quad \lim_{x\to2^+} \frac{x^2}{x^2-4} = \infty$$

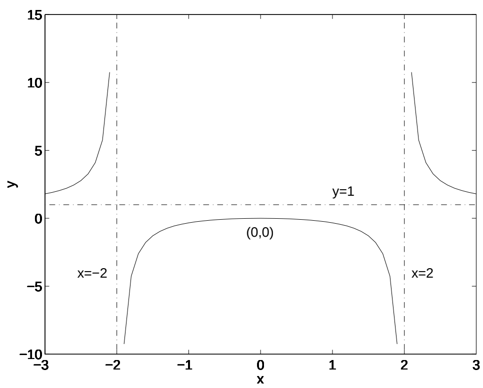
3. $f'(x) = \frac{-8x}{(x^2-4)^2}$; Critical number: $x = 0$
4. Interval of increasing and decreasing: Note: first divide the domain into subintervals according to the location of critical numbers.

	$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$
x	-3	-1	1	3
$f'(x)$	+	+	-	-
$f(x)$	Inc	Inc	Dec	Dec

   Interval of increasing: $(-\infty,-2)\bigcup(-2,0)$; Interval of decreasing: $(0,2)\bigcup(2,\infty)$
5. Local maximum at $x = 0$; local minimum: None
6. $f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3} \neq 0$
   Inflection points: None.
7. Determine where the graph is concave up and concave down and locate any inflection points; Note: first divide the domain into subintervals according to the location where $f''(x) = 0$.

	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
x	-3	0	3
$f''(x)$	+	-	+
Concavity of $f(x)$	Up	Down	Up

   Concave Up: $(-\infty,-2) \cup (2,\infty)$; Concave Down: $(-2,2)$
8. Locate x- and y- intercepts: $(0,0)$
9. graph of the function showing all significant features.



10. Other questions: (Without sketching the graph) Can you find the abs max/min of $f(x)$ on $(-2,2)$, $[-3,0]$, $(-\infty,\infty)$ etc.?