

工資, 經濟 96 學年上學期微積分期中考解答.

Part I:

#1. D #2. B #3. A #4. B #5. B #6. B

#7. B #8. A

#9. 6 #10. 21 #11. 3 #12. $y = -1$

Part II:

#13. ① $f(x) = \frac{1}{2} \ln(e^{2x} e^{\frac{1}{x}} (x^2+1)^{10}) = \frac{1}{2} [\ln e^{2x} + \ln e^{\frac{1}{x}} + \ln (x^2+1)^{10}]$
 $= x + \frac{1}{2x} + 5 \ln(x^2+1)$

$$f'(x) = 1 - \frac{1}{2}x^{-2} + \frac{10x}{x^2+1}$$

② $\frac{d}{dx}(e^{x^3+x}) = e^{x^3+x} \cdot \frac{d}{dx}(x^3+x) = (3x^2+1)e^{x^3+x}$

#14.

① $\frac{d}{dx}(y^2 - x^3) = \frac{d}{dx}(-7) \Rightarrow 2y \cdot \frac{dy}{dx} - 3x^2 = 0 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2y}$

② The slope of the tangent line at $(2, 1)$ is $\left. \frac{dy}{dx} \right|_{(2,1)} = 6$

\therefore tangent line: $\frac{y-1}{x-2} = 6$

#15.

① $\frac{d}{dx} \left(\frac{x+1}{\sqrt{x^2+4}} \right) = \frac{\sqrt{x^2+4} - (x+1) \frac{x}{\sqrt{x^2+4}}}{x^2+4} = \frac{4-x}{(x^2+4)^{\frac{3}{2}}}$

② $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h} \cdot \frac{\sqrt{x+h-1} + \sqrt{x-1}}{\sqrt{x+h-1} + \sqrt{x-1}}$
 $= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h-1} + \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}}$

#16.

① $f(x) = 3x^2 - 3$, $\begin{cases} f'(x) = 0 \Rightarrow x = \pm 1 \\ f'(x) \text{ DNE} \Rightarrow \text{None} \end{cases} \Rightarrow$ critical no. are $x = \pm 1$

② Since the absolute extrema happens on critical point and end point.
Note that -1 & $[0, 2]$.

Just consider $f(1) = 7$... absolute min.

$$f(0) = 9$$

$f(2) = 11$... absolute max.

