Chap 1: Sec. 1.2-Sec. 1.5:

1. Let $f(x) = \begin{cases} |x+2| & \text{for } x \leq 0; \\ 2+x^2 & \text{for } 0 < x < 2; \\ x^3 & \text{for } x \geq 2 \end{cases}$. Find (a) $\lim_{x\to 0^-} f(x)$, (b)$\lim_{x\to 0^+} f(x)$, (c) $\lim_{x\to 2^-} f(x)$, (d)$\lim_{x\to 2^+} f(x)$, (e) $\lim_{x\to 0} f(x)$, (f) $\lim_{x\to 2} f(x)$.
   Ans: (a) 2 (b) 2 (c) 6 (d) 8 (e) 2 (f) DNE

2. Let $f(x) = \begin{cases} cx-2 & \text{for } x \leq 2; \\ cx^2+2 & \text{for } x > 2 \end{cases}$ Find c such that $f(x)$ is continuous. Ans: $c = -2$

3. Determine the intervals on which $f(x) = \ln(1-x^2)$ is continuous.
   Ans: $1 - x^2 > 0$ or $(-1, 1)$

4. Compute (i) $\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x}$ (ii) $\lim_{x\to 1^-} \frac{2x}{x^2-1}$
   Ans: (i) 6 (ii) $-\infty$

Chap 2: Sec. 2.3-Sec. 2.9:

1. Find the tangent line to the curve $y = x^3 - 4x^2 + 2x + 1$ at the point $(1, 0)$.
   Ans: $\frac{dy}{dx} = 3x^2 - 8x + 2$; slope$=-3$; $y - 0 = -3(x-1)$

2. Let $y = e^{x^2} \cdot (x^2 + x + 1) \cdot \sqrt{3x+1}/(x^2 - 1)$. Find $\frac{dy}{dx}$.
   Hint: Take "ln" on both side.

3. The equation $7x^2y^3 - 5xy^2 - 4y = 7$ defines y implicitly as a function of x. Find $\frac{dy}{dx}$.
   Ans:$\frac{14xy^3-5y^2}{4+10xy-21x^2y^2}$

4. Find the derivative of (i) $f(x) = x^{2x}$; (ii) $g(x) = \frac{x^2-x}{3x+1}$; (iii) $h(x) = \ln\sqrt{\frac{3x+1}{5x+2}}$ (assuming $3x+1 > 0$)
   Ans: (i)$2(\ln x + 1)x^{2x}$;(ii) $\frac{(2x-1)(3x+1)-(x^2-x)(3)}{(3x+1)^2} = \frac{(3x-1)(x+1)}{(3x+1)^2}$; (iii)$= \frac{1}{2}(\frac{3}{3x+1} - \frac{5}{5x+2})$

5. Determine if $f(x) = x^7 + 2x^3 - 2006$ is increasing, decreasing or neither. Prove $f(x) = 0$ has exactly one solution. Hint: Sec. 2.9 example 9.1

Chap 3: Sec. 3.1-Sec. 3.8:

1. Estimate $\sqrt[3]{8.02}$ by the method of linear approximation (i.e., by differentials).
   Ans: $f(x) = x^{1/3}$;$f(x) \approx f(8) + f'(8)(x-8)$;$\sqrt[3]{8.02} \approx 2 + \frac{1}{12}(0.02) \approx 2.001667$

2. Find the asymptotes of
   (i) $f(x) = \frac{(3x-1)^2}{9x^2-4}$. Ans: V:$x = 2/3$, $x = -2/3$; H:$y = 1$
   (ii) $f(x) = \frac{(3x-1)^2}{9x^2-1}$.Ans: V:$x = -1/3$; H:$y = 1$
   (iii)$f(x) = \frac{(3x-1)^2}{x-1}$ Ans: V:$x = 1$; H:none; S:$y = 9x + 3$.

3. Let $f(x) = 2x^3 - 3x^2 - 12x$. Find the relative extrema of $f(x)$.
   Ans: local max at $x = -1$; local min at $x = 2$; no abs. max/min

4. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 9x^2 + 12x$ over the interval $[0, 2]$. Ans: Abs max: $f(1) = 5$; abs min: $f(0) = 0$

5. Determine the concavity of $f(x) = 4x^3 - x^4$.
   Ans: Concave up: $(-\infty, 0) \cup (2, \infty)$; Concave down: $(0, 2)$

6. If 300 cm^2 of material is available to make a box with square base and an open top, find the largest possible volume of the box. Explain why your answer is the absolute maximum. Hint:example 6.2

7. Sketch the graph of the continuous function f that satisfies the conditions:

$$\begin{aligned} f''(x) &> 0 \quad \text{if } |x|>2, \quad f''(x)<0 \quad \text{if } |x|<2; \\ f'(0) &= 0, \quad f'(x)>0, \quad \text{if } x<0, \quad f'(x)<0, \quad \text{if } x>0; \\ f(0) &= 1, \quad f(2)=\frac{1}{2}, \quad f(x)>0 \quad \text{for all } x, \text{and } f \text{ is and even function.} \end{aligned}$$

   Ans: The graph looks like $e^{-x^2/8}$

8. An automobile dealer is selling cars at a price of \$12,000. The demand function is $D(p) = 2(15 - 0.001p)^2$, where p is the price of a car. Should the dealer raise or lower the price to increase the revenue? Hint: Find the derivative of the revenue function, $R(p) = p \cdot D(p)$

9. Compute:
   (i) $\lim_{x\to 0}(\frac{1}{\ln(x+1)} - \frac{1}{x})$; (ii) $\lim_{x\to 1^+} \frac{\ln x}{(x-1)^2}$
   Ans (i) Sec 3.2 example 2.7 (ii) Sec 3.2 exercise 38

Chap 4: Sec. 4.2-Sec. 4.8 (Integration Tables), 4.10,

1. Let $f(x) = x + 1$
   (a) Divide the interval $[0, 5]$ into n equal parts, and using right endpoints find an expression for the Riemann sum R_n.
   Hint: $1 + 2 + ... + n = \frac{n(n+1)}{2}$
   (b) Using the answer you got from part(a), calculate $\lim_{n\to\infty} R_n$ (without using antiderivatives).

2. Evaluate the given integral
   (i) $\int x(x+1)^9\,dx$, Hint:$u = x + 1$
   (ii) $\int \frac{dx}{e^x\sqrt{4+e^{2x}}}$. Hint: $u = e^x$, 積分表
   (iii) $\int \frac{\ln x}{x\sqrt{1+\ln x}}\,dx$, Hint:$u = \ln x$
   (iv) $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$. Hint: $u = 1 + x^2$

3. Evaluate the given integral
   (i) $\int \sqrt{x}e^{\sqrt{x}}\,dx$ =?; (ii) $\int \frac{\sqrt{\ln x}}{x}\,dx$ =?; (iii) $\int \frac{x}{x+4}\,dx$ =?; (iv) $\int(\ln x)^2\,dx$ =?;
   Hint: (i) $u = \sqrt{x}$;(ii) $u = \ln x$; (iii) $= x - 4\ln|x + 4|$; (iv) $u = (\ln x)^2$;$dv = dx$

4. Evaluate the given integral
   (i) $\int \frac{\ln x}{x}\,dx$ =?; (ii) $\int \ln(x^2)\,dx$ =?; (iii) $\int \frac{3x}{x^2-3x-4}\,dx$ =?; (iv) $\int \frac{-2x^2+4}{x^3+2x^2+x}\,dx$ =?
   Hint: (i) $u = \ln x$;(ii)$u = \ln x^2$;$dv = dx$; (iii) $x^2 - 3x - 4 = (x - 4)(x + 1)$;
   (iv) $x^3 + 2x^2 + x = x(x + 1)^2$

5. Evaluate the definite integrals:
   (i) $\int_1^4 \sqrt{x}e^{\sqrt{x}}\,dx$ =?; (ii) $\int_1^e (\ln x)^2\,dx$ =?;

6. Evaluate the given integral
   i) $\int_{-\infty}^{\infty} \frac{e^{-x}}{(1+e^{-x})^2}\,dx$ $\int_{-\infty}^{\infty} x^3\,dx$ iii) $\lim_{R\to\infty}\int_{-R}^{R} x^3\,dx$
   Ans: (i) 1 (ii) DIV (iii) 0

7. Determine whether the integral converges or diverges:
   i) $\int_0^1 x^{-1/3}\,dx$ ii) $\int_0^1 x^{-4/3}\,dx$ iii) $\int_1^{\infty} x^{-1/3}\,dx$ iv) $\int_{-1}^1 x^{-1/3}\,dx$
   Ans: (i) 3/2 (ii) DIV (iii) DIV (iv) 0