Calculus I Name:________________________

TA/classroom:____________ Student ID:________________________

Quiz 10

Dec. 26, 2007

1. (5 pts) Use Riemann Sums to compute the given definite integral

$$\int_0^1 x^2\,dx$$

Set $f(x) = x^2$, $\Delta x = \frac{1-0}{N}$ and $x_i = 0 + i\Delta x = \frac{i}{N}$.

Then

$$A_N = \sum_{i=1}^{N} f(x_i)\Delta x = \sum_{i=1}^{N} (\frac{i}{N})^2 \frac{1}{N} = \frac{1}{N^3}\sum_{i=1}^{N} i^2 = \frac{1}{N^3}\frac{N(N+1)(2N+1)}{6}$$

$$\int_0^1 x^2\,dx = \lim_{N\to\infty} \frac{1}{N^3}\frac{N(N+1)(2N+1)}{6} = \frac{1}{3}$$

2. (5 pts) Use the Fundamental Theorem to compute the given definite integral

$$\int_0^1 x^2\,dx = \frac{1}{3}x^3|_0^1 = \frac{1}{3}$$

3. (10 pts) Given $F(x) = \int_x^{x^2} \sqrt{t^2+1}\,dt$, use the Fundamental Theorem to compute $F'(x)$

$$F'(x) = \sqrt{x^4+1}(x^2)' - \sqrt{x^2+1}(x)' = 2x\sqrt{x^4+1}' - \sqrt{x^2+1}.$$

or, let $G(x) = \int_0^x \sqrt{t^2+1}\,dt$, then we have $F(x) = G(x^2) - G(x)$ and, by Fundamental Theorem of Calculus, $G'(x) = \sqrt{x^2+1}$.

Thus

$$F'(x) = (G(x^2) - G(x))' = G'(x^2)(x^2)' - G'(x) = 2x\sqrt{x^4+1} - \sqrt{x^2+1}.$$

- Theorem 1.1 If n is any positive integer and c is any constant, then

$$\sum_{i=1}^{n} c = cn, \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

- Fundamental Theorem of Calculus Part I: If f is continuous on $[a,b]$ and $F(x)$ is any antiderivative of f, then $\int_a^b f(x)dx = F(b) - F(a)$.
- Fundamental Theorem of Calculus, Part II: If f is continuous on $[a,b]$ and $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x)$, on $[a,b]$.