Calculus I Name:_____
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Quiz 3

Oct. 24, 2007

1. (10 pts) Determine the value of a that makes the given function continuous at $x = 0$.

$$f(x) = \begin{cases} ae^x + 2 & \text{if } x < 0 \\ a^5x^5 + a^2x^2 + x - a & \text{if } x \geq 0 \end{cases}$$

Since

$$\begin{aligned} \lim_{x\to 0^-} f(x) &= ae^0 + 2 = a + 2, \\ \lim_{x\to 0^+} f(x) &= -a, \\ f(0) &= -a, \end{aligned}$$

$f(x)$ will be continuous at $x = 0$ if we let $a + 2 = -a$ or $a = -1$.

2. Determine the following limits (answer as appropriate, with a number, $-\infty$, ∞ or does not exist).

- (3 pts) $\lim_{x\to 2^-} \dfrac{x}{2-x}$

$$\lim_{x\to 2^-} \frac{x}{2-x} = +\infty \quad (x \to 2^- \Rightarrow 0 < x < 2; 2 - x > 0 \,\text{and}\, 2 - x \to 0^+)$$

- (3 pts) $\lim_{x\to 2^+} \dfrac{x}{2-x}$

$$\lim_{x\to 2^+} \frac{x}{2-x} = -\infty \quad (x \to 2^+ \Rightarrow x > 2; 2 - x < 0 \,\text{and}\, 2 - x \to 0^-)$$

- (4 pts) $\lim_{x\to +\infty} \dfrac{x}{2-x}$

$$\lim_{x\to\infty} \frac{x}{2-x} = \lim_{x\to\infty} \frac{1}{2/x - 1} = -1$$