

Quiz 4

Oct. 31, 2007

1. (10 pts) Given function $f(x) = \frac{1}{x}$ (for $x \neq 0$), compute the $f'(2)$ by definition

$$(f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}).$$

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{2+h} - \frac{1}{2} \right) \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{2 - (2+h)}{(2+h)2} \right) \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{-h}{(2+h)2} \right) \\ &= -\frac{1}{4} \end{aligned}$$

2. (10 pts) Given function $f(x) = \sqrt{x+1}$ (for $x \geq -1$), compute the $f'(x)$ by definition

$$(f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}).$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \\ &= \lim_{h \rightarrow 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \\ &= \frac{1}{2\sqrt{x+1}} \end{aligned}$$