Quiz 6
Nov. 14, 2007

1. (10 pts) Find the derivative of

$$f(x) = \ln \sqrt{x^{10}e^{2x}(x^2 + 1)}$$

   Ans: Since

$$\begin{aligned}
f(x) &= \ln\left(x^{10}e^{2x}(x^2 + 1)\right)^{1/2} \\
&= \frac{1}{2}\ln\left(x^{10}e^{2x}(x^2 + 1)\right) \\
&= \frac{1}{2}[\ln x^{10} + \ln e^{2x} + \ln(x^2 + 1)] \\
&= \frac{1}{2}[10\ln x + 2x + \ln(x^2 + 1)],
\end{aligned}$$

$$f'(x) = \frac{1}{2}[10\frac{1}{x} + 2 + \frac{1}{x^2 + 1}\cdot(x^2 + 1)'] = \frac{5}{x} + 1 + \frac{x}{x^2 + 1}.$$

2. (10 pts) Given the curve $y^3 - x^2 = -3$. Find $\frac{dy}{dx}$ implicitly. What is the equation of the tangent line at $(2, 1)$.

   Ans: Apply $\frac{d}{dx}$ to the equation, $y^3 - x^2 = -3$, we have

$$3y^2\frac{dy}{dx} - 2x = 0,$$

   or

$$\frac{dy}{dx} = \frac{2x}{3y^2}.$$

   Since the point $(2, 1)$ is on the curve, slope of the tangent line at $(2, 1)$ is given by

$$m = \frac{dy}{dx}\Big|_{(2,1)} = \frac{2\cdot 2}{3} = \frac{4}{3}.$$

   The equation of the tangent line is given by

$$y - 1 = \frac{4}{3}(x - 2).$$