Calculus I Name:________________________
TA/classroom:____________ Student ID:________________________

Quiz 7

Dec. 5, 2007

1. (8 pts) Estimate $\sqrt{4.02}$ by the method of linear approximation.

   Let $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2}x^{-1/2}$. Since $f(4) = 2$ and $4 \approx 4.02$, we are looking for a linear approxmiation near $x = 4$.

   $L(x) = f(4) + f'(4)(x-4) = 2 + \frac{1}{2}\frac{1}{2}(x-4) = 2 + \frac{1}{4}(x-4)$.

   So $L(4.02) = 2 + \frac{1}{4}(4.02-4) = 2.005 \approx \sqrt{4.02}$

2. (6 pts) Compute

   $$\lim_{x\to 0} \frac{e^x - 1}{x^2}$$

   Note that $\lim_{x\to 0} e^x - 1 = 0$ and $\lim_{x\to 0} x^2 = 0$. The limit has an indeterminate form $\frac{0}{0}$ and we can apply the L'Hôpital's Rule.

   $$\lim_{x\to 0} \frac{e^x - 1}{x^2} = \lim_{x\to 0} \frac{e^x}{2x} = DNE$$

   Note that $\lim_{x\to 0^-} \frac{e^x}{2x} = -\infty$ and $\lim_{x\to 0^+} \frac{e^x}{2x} = \infty$.

3. (6 pts) Compute

   $$\lim_{x\to 0^+} x^x$$

   Note that the limit has an indeterminate form 0^0 and we can apply the L'Hôpital's Rule. Let $y = x^x$, so that $\ln y = \ln x^x = x \ln x \quad (x > 0)$. Now consider the limit

   $$\begin{aligned}\lim_{x\to 0^+} \ln y &= \lim_{x\to 0^+} x \ln x \\ &= \lim_{x\to 0^+} \frac{\ln x}{1/x} \\ &= \lim_{x\to 0^+} \frac{1/x}{-1/x^2} \quad \text{(By L'Hôpital's Rule.)} \\ &= \lim_{x\to 0^+} -x \\ &= 0\end{aligned}$$

   Thus

   $$\begin{aligned}\lim_{x\to 0^+} x^x &= \lim_{x\to 0^+} e^{\ln y} \\ &= e^{\lim_{x\to 0^+} \ln y} \quad (e^x \text{ is a continuous function.}) \\ &= e^0 \\ &= 1\end{aligned}$$