Calculus I Quiz 1 Oct. 14, 2009

Name:__________ Student ID number: __________

## Part I: Problem 1-5 選擇及填充-No Partial Credit

C 1. (5 pts) $\lim_{x\to-\infty} \dfrac{x^2-x-1}{2x^2-7} = ?$

A) $\infty$ B) $-\infty$,

C) $1/2$, D) $-1/2$

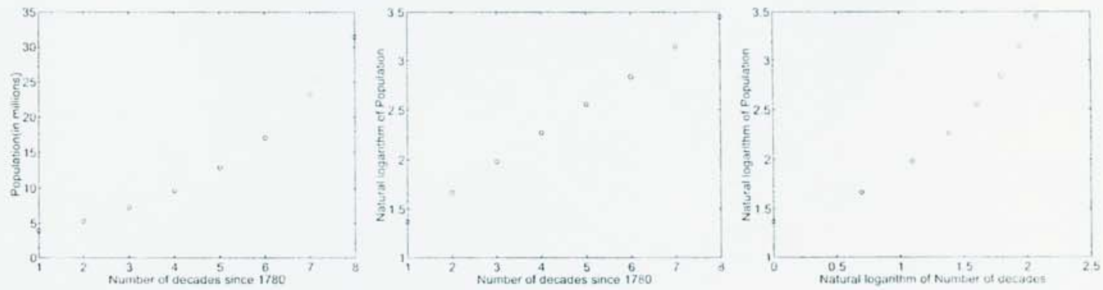
A 2. (5 pts) $\lim_{x\to\infty} \sqrt{x^2+1} = ?$

A) $\infty$ B) $-\infty$,

C) 1, D) $-1$

A 3. (5 pts) The population of the United States from 1790 to 1860 was shown in the table below.

| Year | 1790 | 1800 | 1810 | 1820 | 1830 | 1840 | 1850 | 1860 |
|---|---|---|---|---|---|---|---|---|
| Population | 3,929,214 | 5,308,483 | 7,239,881 | 9,638,453 | 12,866,020 | 17,069,453 | 23,191,876 | 31,443,321 |




Determine if the population of the United States from 1790 to 1860 was increasing exponentially or as a polynomial.

A) exponentially ($y = ae^{bx}$). B) as a polynomial ($y = bx^n$).

4. (10 pts) Match the curves in the figure to the functions?

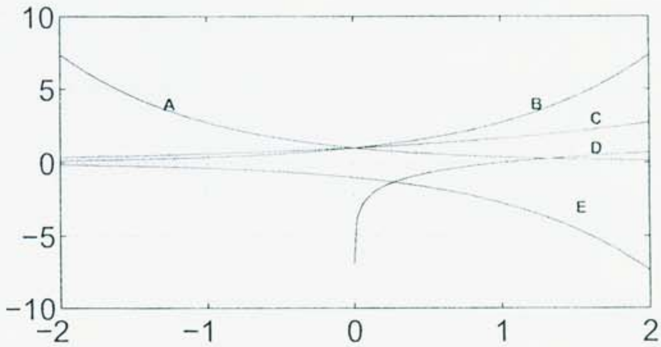
$e^x =$ B,

$e^{-x} =$ A,

$e^{x/2} =$ C,

$-e^x =$ E,

$\ln x =$ D,



Name:______________ Student ID number: ______________

5. (10 pts) Identify the limits from the graph of $f(x)$

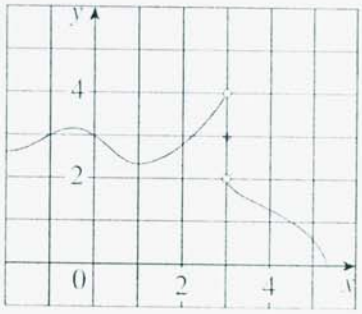
$\lim_{x \to 0} f(x) =$ ___ 3 ___,

$\lim_{x \to 3^+} f(x) =$ ___ 2 ___,

$\lim_{x \to 3^-} f(x) =$ ___ 4 ___,

$\lim_{x \to 3} f(x) =$ ___ DNE ___,

$f(3) =$ ___ 3 ___.




Part II: Problem 6-13 計算及證明題

6. (10 pts) Find the domain of

(a) $f(x) = \frac{1}{1-e^x}$.

$1 - e^x \neq 0$

$e^x \neq 1$

$x \neq 0 \quad \Rightarrow D = \{x \in \mathbb{R} \mid x \neq 0\}$

(b) $g(x) = \ln(2 + \ln(x))$.

$\ln x \Rightarrow x > 0$

$\ln(2 + \ln x) \Rightarrow 2 + \ln x > 0$

$\ln x > -2, \ x > e^{-2} \quad \Rightarrow D = \{x \in \mathbb{R} \mid x > e^{-2}\}$

7. (10 pts) Find the inverse function of

(a) $f(x) = \dfrac{4x-1}{2x+3}$.

$y = \dfrac{4x-1}{2x+3}$

$(2x+3)y = 4x - 1$

$2xy - 4x = -3y - 1$

$f^{-1}(y) = x = \dfrac{-3y-1}{2y-4}$

$\therefore f^{-1}(x) = \dfrac{-3x-1}{2x-4}, \ x \neq 2$

(b) $g(x) = \ln(x+3)$.

$y = \ln(x+3)$

$e^y = x + 3$

$f^{-1}(y) = x = e^y - 3$

$\therefore f^{-1}(x) = e^x - 3$

8. (10 pts) Show that $f(x) = \cos x - x$ has a zero in $(0, 1)$ (use Intermediate Value Theorem)

$\because$ $x$ and $\cos x$ are continuous for all $x \in \mathbb{R}$

$\therefore$ $\cos x - x$ is continuous on $(-\infty, \infty)$

$\Rightarrow$ $\cos x - x$ is continuous on $[0, 1]$

$\because$ $f(0) = \cos 0 - 0 = 1 > 0$

$f(1) = \cos 1 - 1 < 0$

by IVT, $\exists\, c \in (0, 1) \ni f(c) = 0$. done.

9. (10 pts) Evaluate the limit, if it exists.

(a) $\lim_{x \to 7} \frac{\sqrt{x+2} - 3}{x - 7}$.

$$= \lim_{x \to 7} \frac{\sqrt{x+2} - 3}{x - 7} \times \frac{\sqrt{x+2} + 3}{\sqrt{x+2} + 3} = \lim_{x \to 7} \frac{\cancel{x+2-9}}{\cancel{(x-7)}(\sqrt{x+2} + 3)}$$

$$= \lim_{x \to 7} \frac{1}{\sqrt{x+2} + 3} = \frac{1}{3+3} = \frac{1}{6}$$

(b) $\lim_{t \to 1} (\frac{1}{t} - \frac{1}{t^2 + t})$.

$$= \lim_{t \to 1} \frac{1}{t} - \lim_{t \to 1} \frac{1}{t^2 + 1}$$

$$= 1 - \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Name:____________ Student ID number: ____________

10. (10 pts) Determine whether $f'(0)$ exists. (Hint: the definition of derivative and the Squeeze Theorem)

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

$f'(0) = \lim\limits_{h \to 0} \dfrac{f(h) - f(0)}{h}$

$= \lim\limits_{h \to 0} \dfrac{h^2 \sin \frac{1}{h} - 0}{h}$

$= \lim\limits_{h \to 0} h \sin \frac{1}{h}$

for $h > 0$

$-1 \le \sin \frac{1}{h} \le 1$

$\Rightarrow -h \le h \sin \frac{1}{h} \le h$

$\because \lim\limits_{h \to 0^+} h = 0 = \lim\limits_{h \to 0^+} (-h)$

$\therefore \lim\limits_{h \to 0^+} h \sin \frac{1}{h} = 0$

for $h < 0$

$-1 \le \sin \frac{1}{h} \le 1$

$\Rightarrow h \le h \sin \frac{1}{h} \le -h$

$\because \lim\limits_{h \to 0^-} (-h) = 0 = \lim\limits_{h \to 0^-} h$

$\therefore \lim\limits_{h \to 0^-} h \sin \frac{1}{h} = 0$

$\because \lim\limits_{h \to 0^+} h \sin \frac{1}{h} = 0 = \lim\limits_{h \to 0^-} h \sin \frac{1}{h}$

$\therefore \lim\limits_{h \to 0} h \sin \frac{1}{h} = 0 \Rightarrow f'(0) = 0$

or

Let $0 \le |\sin \frac{1}{h}| \le 1$

$\Rightarrow 0 \le |h \sin \frac{1}{h}| \le |h|$

$\because \lim\limits_{h \to 0} 0 = 0$

$\lim\limits_{h \to 0} |h| = 0$

$\therefore \lim\limits_{h \to 0} |h \sin \frac{1}{h}| = 0$

$\Rightarrow \lim\limits_{h \to 0} h \sin \frac{1}{h} = 0$

$\Rightarrow f'(0) = 0$

11. (10 pts) Let $f(x) = \begin{cases} x^2 + 1, & x \le 1, \\ mx + b, & x > 1. \end{cases}$ Find the value of m and b that make $f$ differentiable at $x = 1$.

Continuity:

$\lim\limits_{x \to 1^+} f(x) = 1 + 1 = 2$

$\lim\limits_{x \to 1^-} f(x) = m + b$

$\Rightarrow m + b = 2$

Differentiability:

$\lim\limits_{h \to 0^+} \dfrac{f(1+h) - f(1)}{h}$

$= \lim\limits_{h \to 0^+} \dfrac{1 + 2h + h^2 + 1 - 2}{h}$

$= \lim\limits_{h \to 0^+} 2 + h = 2$

$\lim\limits_{h \to 0^-} \dfrac{f(1+h) - f(1)}{h}$

$= \lim\limits_{h \to 0^-} \dfrac{m + mh + b - 2}{h}$

$= \lim\limits_{h \to 0^-} m \quad (\because m + b = 2)$

$= m$

$\Rightarrow \begin{cases} m = 2 \\ b = 0 \end{cases}$

12. (5 pts) Find the derivative of

$$y = \frac{x^2 - 2\sqrt{x}}{x}$$

$y = x - 2x^{-\frac{1}{2}}$

$\Rightarrow y' = 1 - 2(-\frac{1}{2}) x^{-\frac{3}{2}}$

$= 1 + x^{-\frac{3}{2}}$