Calculus I Quiz 2 — Dec. 9, 2009

Name: ____________ Student ID number: ____________

1. (30 pts; 2 pts for each problem; **No partial credit**) Find the general antiderivative of the given function ($f(x) = F'(x)$).

   (a) $f(x) = 1$, $F(x) =$ x $+C$

   (b) $f(x) = x^2$, $F(x) =$ $\frac{1}{3}x^3$ $+C$

   (c) For $x \geq 0$, $f(x) = \sqrt{x}$, $F(x) =$ $\frac{2}{3}x^{\frac{3}{2}}$ $+C$

   (d) $f(x) = x^{1/3}$, $F(x) =$ $\frac{3}{4}x^{\frac{4}{3}}$ $+C$

   (e) $f(x) = e^x$, $F(x) =$ e^x $+C$

   (f) $f(x) = e^{4x}$, $F(x) =$ $\frac{1}{4}e^{4x}$ $+C$

   (g) $f(x) = 2^x$, $F(x) =$ $\frac{2^x}{\ln 2}$ $+C$

   (h) For $x \neq 0$, $f(x) = \frac{1}{x}$, $F(x) =$ $\ln|x|$ $+C$

   (i) For $x \neq 0$, $f(x) = \frac{1}{2x}$, $F(x) =$ $\frac{1}{2}\ln|x|$ (or $\frac{1}{2}\ln|2x|$) $+C$

   (j) $f(x) = \sin x$, $F(x) =$ $-\cos x$ $+C$

   (k) $f(x) = \cos x$, $F(x) =$ $\sin x$ $+C$

   (l) $f(x) = \sec^2 x$, $F(x) =$ $\tan x$ $+C$

   (m) $f(x) = \sec x \tan x$, $F(x) =$ $\sec x$ $+C$

   (n) $f(x) = \frac{1}{1+x^2}$, $F(x) =$ $\tan^{-1} x$ $+C$

   (o) For $x \neq 0$, $f(x) = \frac{x^2+1}{x}$, $F(x) =$ $\frac{1}{2}x^2 + \ln|x|$ $+C$

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2. (5 pts) Given that $f'(x) = 1 - 6x$ and $f(0) = 8$, find $f(x)$.

$f(x) = x - 3x^2 + C$

$f(0) = 0 - 0 + C = 8 \Rightarrow c = 8$

$\therefore f(x) = x - 3x^2 + 8$

3. (5 pts) Express the limit as a definite integral on the given interval.

$$\lim_{n\to\infty} \sum_{i=1}^{n} 2x_i \ln(1 + x_i^2)\,\Delta x, \quad [2, 6], \quad (x_i = 2 + i\Delta x, \Delta x = (6-2)/n)$$

$$\int_2^6 2x \ln(1 + x^2)\,dx$$

4. (10 pts) Find the derivative of

- $\displaystyle\int_0^x \sqrt{t}\sin t\,dt,\ x \ge 0,$

$$\frac{d}{dx}\left(\int_0^x \sqrt{t}\sin t\,dt\right) = \sqrt{x}\sin x \cdot \frac{d}{dx}(x) = \sqrt{x}\sin x$$

- $\displaystyle\int_{x^2}^{x^4} \sqrt{t}\sin t\,dt,\ x \ge 0.$

$$\frac{d}{dx}\left(\int_{x^2}^{x^4} \sqrt{t}\sin t\,dt\right) = \sqrt{x^4}\sin x^4 \cdot \frac{d}{dx}(x^4) - \sqrt{x^2}\sin x^2 \cdot \frac{d}{dx}(x^2)$$

$$= (x^2 \sin x^4)(4x^3) - (x \sin x^2)(2x)$$

$$= 4x^5 \sin x^4 - 2x^2 \sin x^2$$

5. (10 pts) Evaluate the definite integral $\displaystyle\int_0^1 xe^{x^2}\,dx$

Let $u = x^2$ $\qquad x = 1 \Rightarrow u = 1$

$du = 2x\,dx$, $x\,dx = \frac{du}{2}$ $\qquad x = 0 \Rightarrow u = 0$

$$\therefore \int_0^1 xe^{x^2}dx = \int_0^1 e^u \left(\frac{du}{2}\right) = \left.\frac{e^u}{2}\right|_0^1$$

$$= \frac{1}{2}(e - 1)$$

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6. (10 pts) Evaluate the definite integral $\int_{-1}^{2} x - 2|x|\,dx$

$$\int_{-1}^{2} x-2|x|\,dx = \int_0^2 (x-2x)\,dx + \int_{-1}^{0} (x-2(-x))\,dx$$

$$= \int_0^2 -x\,dx + \int_{-1}^{0} 3x\,dx = -\frac{1}{2}x^2\Big|_0^2 + \frac{3}{2}x^2\Big|_{-1}^{0}$$

$$= (-2-0)+\left(0-\frac{3}{2}\right) = \frac{-7}{2}$$

7. (10 pts) Evaluate $\int \ln x\,dx$

Let $u=\ln x$ $dv=dx$

$du=\frac{1}{x}dx$ $v=x$

$$\int \ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}\,dx$$

$$= x\ln x - \int 1\,dx$$

$$= x\ln x - x + C$$

8. (10 pts) Evaluate $\int \sin^4 x\cos^3 x\,dx$

Let $u=\sin x$, $du=\cos x\,dx$

$$\int \sin^4 x\cos^3 x\,dx = \int \sin^4 x(1-\sin^2 x)\cos x\,dx = \int u^4(1-u^2)\,du$$

$$= \int u^4-u^6\,du = \frac{1}{5}u^5 - \frac{1}{7}u^7 + C = \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$$

9. (10 pts) Evaluate $\int \frac{1}{\sqrt{1+x^2}}\,dx$. (Hint: $\int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta| + C$)

Let $x=\tan\theta$, $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

$dx=\sec^2\theta\,d\theta$, $\sqrt{1+x^2}=\sqrt{1+\tan^2\theta}=\sqrt{\sec^2\theta}=|\sec\theta|=\sec\theta$

$(\because \sec\theta>0)$

$$\int\frac{1}{\sqrt{1+x^2}}\,dx = \int\frac{1}{\sec\theta}\cdot\sec^2\theta\,d\theta = \int\sec\theta\,d\theta = \ln|\sec\theta+\tan\theta| + C$$

$$= \ln\left|\sqrt{1+x^2}+x\right| + C$$

$\sec\theta=\sqrt{1+x^2}$

$\tan\theta=x$