Revised November 7, 2009
Steps to determine the local and global extrema of a given function

1. Find all critical points ($f'(c) = 0$ or DNE)
2. Find the endpoints of the domain of f
3. Use the first derivative test to identify the local extrema.

- Example 1: $f(x) = 3x^4 + 4x^3 - 6x^2 - 12x + 12$
  1. Domain of the function $= (-\infty.\infty)$ (no endpoints)
  2. $f'(x) = 12x^3 + 12x^2 - 12x - 12 = 12(x+1)^2(x-1)$;
     Critical numbers: $x = 1$ and $x = -1$.
  3. Interval of increasing and decreasing: Note: first divide the domain into subintervals according to the location of critical numbers.

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
x	-2	0	2
$f'(x)$	-	-	+
$f(x)$	Dec	Dec	Inc

  4. Local maximum: None; local minimum at $x = 1$ (1st derivative test)
  5. Global (abs) maximum: None ($\lim_{x\to\infty} f(x) = \infty$); global minimum at $x = 1$:
     Since $f(x)$ is a polynomial, it is continuous on $(-\infty, \infty)$. Since the function is decreasing to the left of 1 and increasing to the right of 1.
  6. Other questions: (Without sketching the graph) Can you find the abs max/min of $f(x)$ on $[-2, 2]$, and $[-2, 0]$? How do you know that $f(x)$ has no x-intercept (or $f(x) \neq 0$)?

- Example 2: $f(x) = 3x^4 + 4x^3 - 6x^2 - 12x + 12$, $[-2, 2]$
  1. Domain of the function $= [-2, 2]$: Endpoints: -2, 2
  2. $f'(x) = 12x^3 + 12x^2 - 12x - 12 = 12(x+1)^2(x-1)$;
     Critical numbers: $x = 1$ and $x = -1$ (All critical points are in the domain).
  3. Interval of increasing and decreasing: Note: first divide the domain into subintervals according to the location of critical numbers.

	$(-2, -1)$	$(-1, 1)$	$(1, 2)$
x	-1.5	0	1.5
$f'(x)$	-	-	+
$f(x)$	Dec	Dec	Inc

  4. Local maxima at $x = -2$ and $x = 2$; local minimum at $x = 1$ (1st derivative test)
  5. End points: $f(-2) = 28$; $f(2) = 44$; Critical points: $f(-1) = 17$; $f(1) = 1$
     Global (abs) maximum at $x = 2$; global minimum at $x = 1$

- Example 3: $f(x) = 3x^4 + 4x^3 - 6x^2 - 12x + 12$, $[0, 2)$

  1. Domain of the function $= [0, 2)$: Endpoints: 0. (Note 2 is not in the domain)
  2. $f'(x) = 12x^3 + 12x^2 - 12x - 12 = 12(x + 1)^2(x - 1)$;
     Critical numbers: $x = 1$. (-1 is not in the domain).
  3. Interval of increasing and decreasing: Note: first divide the domain into subintervals according to the location of critical numbers.

	$(0,1)$	$(1,2)$
x	.5	1.5
$f'(x)$	-	+
$f(x)$	Dec	Inc

  4. Local maxima at $x = 0$; local minimum at $x = 1$ (1st derivative test)
  5. Critical points: $f(1) = 1$; Endpoints: $f(0) = 12$, $\lim_{x\to 2} f(x) = 44$
     Global (abs) maximum: None ; global minimum at $x = 1$