

Steps to sketch the curve of a given function (and related problems):

- Example 1  $f(x) = 3x^4 + 4x^3 - 6x^2 - 12x + 12$ :

1. Domain of the function =  $(-\infty, \infty)$
2. Horizontal and Vertical Asymptotes: None ( $f(x)$  is a polynomial).
3.  $f'(x) = 12x^3 + 12x^2 - 12x - 12 = 12(x+1)^2(x-1)$ ;  
Critical numbers:  $x = 1$  and  $x = -1$ .
4. Interval of increasing and decreasing: Note: first divide the domain into subintervals according to the location of critical numbers.

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
$x$	-2	0	2
$f'(x)$	-	-	+
$f(x)$	Dec	Dec	Inc

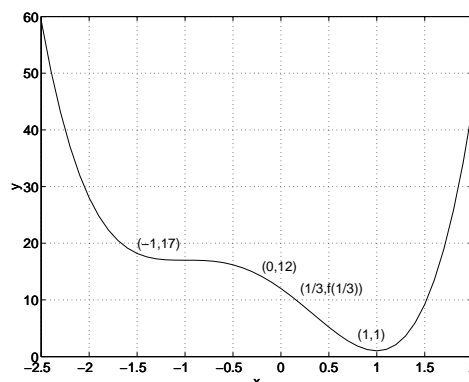
Interval of increasing:  $(1, \infty)$ ; Interval of decreasing:  $(-\infty, 1)$

5. Local maximum: None; local minimum at  $x = 1$  (1st derivative test)
6.  $f''(x) = 36x^2 + 24x - 12 = 12(x+1)(3x-1)$ ;
7. Determine where the graph is concave up and concave down: Note: first divide the domain into subintervals according to the location where  $f''(x) = 0$ .

	$(-\infty, -1)$	$(-1, 1/3)$	$(1/3, \infty)$
$x$	-2	0	1
$f''(x)$	+	-	+
Concavity of $f(x)$	Up	Down	Up

Concave Up:  $(-\infty, -1) \cup (1/3, \infty)$ ; Concave Down:  $(-1, 1/3)$

8. Inflection points:  $x = -1$  and  $x = 1/3$ . (by table above)
9. Locate x- and y- intercepts: No x-intercept; y-intercept:  $(0, 12)$
10. graph of the function showing all significant features.



11. Other questions: (Without sketching the graph) Can you find the abs max/min of  $f(x)$  on  $[-2, 2]$ ,  $[-2, 0]$  and  $(-\infty, \infty)$ ? How do you know that  $f(x)$  has no x-intercept (or  $f(x) \neq 0$ )?

- Given  $f(x) = \frac{x^2}{x^2-4}$ :

1. Domain of the function =  $\{x \in \mathbb{R} | x \neq \pm 2\}$

2. Horizontal and Vertical Asymptotes:

H-asymptote:  $\lim_{x \rightarrow \pm\infty} \frac{x^2}{x^2-4} = \lim_{x \rightarrow \pm\infty} \frac{1}{1-4/x^2} = 1 \Rightarrow y = 1.$

V-asymptotes:  $x = -2$  and  $x = 2$

$$\begin{aligned} \lim_{x \rightarrow -2^-} \frac{x^2}{x^2-4} &= \infty & \lim_{x \rightarrow -2^+} \frac{x^2}{x^2-4} &= -\infty \\ \lim_{x \rightarrow 2^-} \frac{x^2}{x^2-4} &= -\infty & \lim_{x \rightarrow 2^+} \frac{x^2}{x^2-4} &= \infty \end{aligned}$$

3.  $f'(x) = \frac{-8x}{(x^2-4)^2}$ ; Critical number:  $x = 0$

4. Interval of increasing and decreasing: Note: first divide the domain into subintervals according to the location of critical numbers.

	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
$x$	-3	-1	1	3
$f'(x)$	+	+	-	-
$f(x)$	Inc	Inc	Dec	Dec

Interval of increasing:  $(-\infty, -2) \cup (-2, 0)$ ; Interval of decreasing:  $(0, 2) \cup (2, \infty)$

5. Local maximum at  $x = 0$ ; local minimum: None

6.  $f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3} \neq 0$

Inflection points: None.

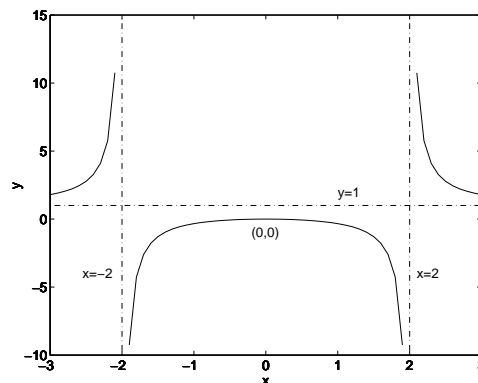
7. Determine where the graph is concave up and concave down and locate any inflection points; Note: first divide the domain into subintervals according to the location where  $f''(x) = 0$ .

	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
$x$	-3	0	3
$f''(x)$	+	-	+
Concavity of $f(x)$	Up	Down	Up

Concave Up:  $(-\infty, -2) \cup (2, \infty)$ ; Concave Down:  $(-2, 2)$

8. Locate x- and y- intercepts:  $(0, 0)$

9. graph of the function showing all significant features.



10. Other questions: (Without sketching the graph) Can you find the abs max/min of  $f(x)$  on  $(-2, 2)$ ,  $[-3, 0]$ ,  $(-\infty, \infty)$  etc.?