

• **Limits and Continuity:**

1. Let $f(x) = \begin{cases} |x+2| & \text{for } x \leq 0; \\ 2+x^2 & \text{for } 0 < x < 2; \\ x^3 & \text{for } x \geq 2 \end{cases}$.

Find (a) $\lim_{x \rightarrow 0^-} f(x)$, (b) $\lim_{x \rightarrow 0^+} f(x)$, (c) $\lim_{x \rightarrow 2^-} f(x)$, (d) $\lim_{x \rightarrow 2^+} f(x)$, (e) $\lim_{x \rightarrow 0} f(x)$, (f) $\lim_{x \rightarrow 2} f(x)$.

2. Compute $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{9x^2}$

3. Let $f(x) = \begin{cases} cx - 2 & \text{for } x \leq 2; \\ cx^2 + 2 & \text{for } x > 2 \end{cases}$. Find c such that $f(x)$ is continuous.

4. Determine the intervals on which $f(x) = \ln(1 - x^2)$ is continuous.

5. (i) $\lim_{x \rightarrow 0} \frac{\sqrt{x+9}-3}{x}$; (ii) $\lim_{x \rightarrow 1^-} \frac{2x}{x^2-1}$; (iii) $\lim_{x \rightarrow \infty} \frac{x+\sin x}{4x+999}$

• **Differentiation:**

1. Find the tangent line to the curve $y = x^3 - 4x^2 + 2x + 1$ at the point $(1, 0)$.

2. Let $y = e^{x^2} \sin(x^2 + x + 1) \cdot \sqrt{3x+1}/(x^2 - 1)$. Find $\frac{dy}{dx}$.

3. The equation $7x^2y^3 - 5xy^2 - 4y = 7$ defines y implicitly as a function of x . Find $\frac{dy}{dx}$.

4. Find the derivative of (i) $f(x) = x^{2x}$; (ii) $g(x) = \frac{x^2-x}{3x+1}$; (iii) $h(x) = \ln \sqrt{\frac{3x+1}{5x+2}}$ (assuming $3x+1 > 0$)

5. Compute $\frac{d}{dx} \cos^{-1}(2x^3)$

6. Determine if $f(x) = x^7 + 2x^3 - 2006$ is increasing, decreasing or neither. Prove $f(x) = 0$ has exactly one solution.

• **Application of Differentiation:**

1. Estimate $\tan((\pi/4) + 0.05)$ by the method of linear approximation (i.e., by differentials).

2. Compute $\lim_{x \rightarrow 1^+} \frac{\ln x}{(x-1)^2}$

3. Find the asymptotes of

(i) $f(x) = \frac{(3x-1)^2}{9x^2-4}$. (ii) $f(x) = \frac{(3x-1)^2}{9x^2-1}$. (iii) $f(x) = \frac{(3x-1)^2}{x-1}$

4. Let $f(x) = 2x^3 - 3x^2 - 12x$. Find the relative extrema of $f(x)$.

5. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 9x^2 + 12x$ over the interval $[0, 2]$.

6. Determine the concavity of $f(x) = 4x^3 - x^4$.

7. If 300 cm^2 of material is available to make a box with square base and an open top, find the largest possible volume of the box. Explain why your answer is the absolute maximum.

8. Sketch the graph of the continuous function f that satisfies the conditions:

$$\begin{aligned} f''(x) &> 0 & \text{if } |x| > 2, & & f''(x) < 0 & \text{if } |x| < 2; \\ f'(0) &= 0, & f'(x) > 0, & \text{if } x < 0, & f'(x) < 0, & \text{if } x > 0; \\ f(0) &= 1, & f(2) = \frac{1}{2}, & & f(x) > 0 & \text{for all } x, \text{ and } f \text{ is an even function.} \end{aligned}$$

9. (i) $\lim_{x \rightarrow 0} \left(\frac{1}{\ln(x+1)} - \frac{1}{x} \right)$; (ii) $\lim_{x \rightarrow 0^+} (\cos x)^{1/x}$; (iii) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x$;

• **Integration:**

- Let $f(x) = x + 1$
 - Divide the interval $[0, 5]$ into n equal parts, and using right endpoints find an expression for the Riemann sum R_n .
 - Using the answer you got from part(a), calculate $\lim_{n \rightarrow \infty} R_n$ (without using antiderivatives).

2. Find the derivatives of $G(x) = \int_0^{x^2} \sqrt{1+t^4} dt$. It is not necessary to simplify your answer:

3. Let f be continuous and define F by

$$F(x) = \int_0^x [t^2 \int_1^t f(u) du] dt.$$

Find $F'(x)$ and $F''(x)$.

4. Evaluate the given integral

$$(i) \int x(x+1)^9 dx, (ii) \int \frac{\cos \theta}{\sin^2 \theta - 2 \sin \theta - 8} d\theta. (iii) \int \frac{dx}{e^x \sqrt{4+e^{2x}}}.$$

$$(iv) \int \frac{\ln x}{x\sqrt{1+\ln x}} dx, (v) \int \frac{x^3}{\sqrt{x^2+1}} dx. (vi) \int_{-\infty}^{\infty} \frac{e^{-x}}{(1+e^{-x})^2} dx$$

5. Evaluate the given integral

$$(i) \int \sqrt{x} e^{\sqrt{x}} dx; (ii) \int \frac{\sqrt{\ln x}}{x} dx; (iii) \int \frac{x}{x+4} dx; (iv) \int (\ln x)^2 dx;$$

6. Evaluate the given integral

$$(i) \int \frac{\ln x}{x} dx; (ii) \int \ln(x^2) dx; (iii) \int \frac{3x}{x^2-3x-4} dx; (iv) \int \frac{-2x^2+4}{x^3+2x^2+x} dx$$

7. Evaluate the definite integrals:

$$(i) \int_1^4 \sqrt{x} e^{\sqrt{x}} dx; (ii) \int_1^e (\ln x)^2 dx;$$

8. Evaluate the given integral

$$(i) \int \sec^3 t dt; (ii) \int \sec t dt;$$

9. (a) $\int_{-\infty}^{\infty} x^3 dx$ (b) $\lim_{R \rightarrow \infty} \int_{-R}^R x^3 dx$

10. Determine whether the integral converges or diverges:

$$(i) \int_0^1 x^{-1/3} dx; (ii) \int_0^1 x^{-4/3} dx; (iii) \int_1^{\infty} x^{-1/3} dx; (iv) \int_{-1}^1 x^{-1/3} dx$$

• Application of Integration

- Find the region bounded by the parabola $x = 2 - y^2$ and the line $y = x$.
- A solid is formed by revolving the circular disk $(x-5)^2 + y^2 = 4$ about the y -axis. Set up, **but do not evaluate**, a definite integral which give the volume of the solid.
- Let Ω be the region bounded by $y = \sec x$, $x = 0$, $x = \frac{\pi}{4}$ and $y = 0$. Find integrals represent the volume of the solids generated by Ω about (a) x-axis, (b) y-axis, (c) $y = -1$, (d) $x = -1$. (**Don't evaluate the integrals**)
- Set up a definite integral for the arc length of an ellipse $x^2 + 4y^2 = 4$.
- Set up the integral for the surface area of the surface of revolution. $y = e^x$, $0 \leq x \leq 1$, revolved about x-axis.
- Find the area of the surface generated by revolving the curve $y = \cosh x = \frac{e^x + e^{-x}}{2}$, $x \in [0, \ln 2]$ about the x-axis.
- Let $f(x) = \frac{3}{64} x \sqrt{16 - x^2}$ for $0 \leq x \leq 4$ and $f(x) = 0$ for other values of x .
 - Verify that f is a probability density function.
 - Find $P(X < 2)$.