Name: ______________________________

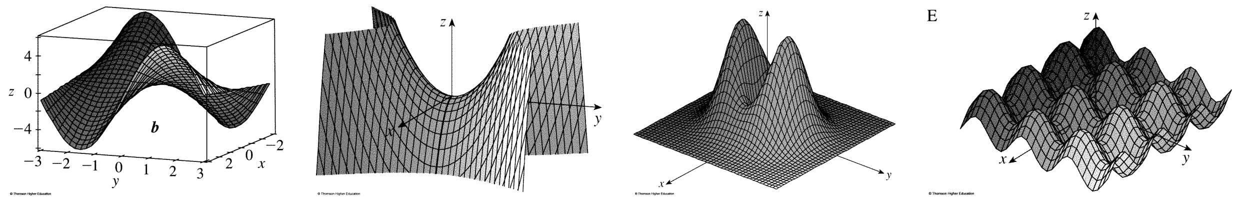
Student ID number: ____________________

TA/classroom:________________________

Guidelines for the test:

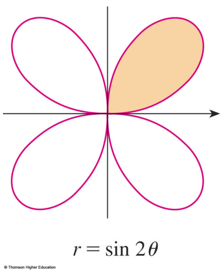
- Put your name or student ID number on every page.
- There are 11 problems
- The exam is closed book; calculators are not allowed.
- For problem-solving (計算與證明題) problems, please show all work, unless instructed otherwise. Partial credit will be given only for work shown. Print as legibly as possible - correct answers may have points taken off, if they're illegible.
- **Mark the final answer**.

1. (5 pts; No Partial Credits) Match the function $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$ with the graphs



(A) (B) (C) (D)

2. (10 pts) Find the area of the region enclosed by the curve $r = \sin 2\theta$, $0 \le \theta \le \pi/2$.



$r = \sin 2\theta$

3. (15 pts)

   (a) Given that $\mathbf{r}(t) =< e^{2t}, t^2 - t, \cos 2t >$, calculate

   - (2 pts) $\lim_{t \to 0} \mathbf{r}(t) =$

   - (4 pts) $\int \mathbf{r}(t)\, dt =$

   (b) Given the position function $\mathbf{r}(t) =< \sin 2t, \cos 2t, t >$,

   - (2 pts) find the velocity, $\mathbf{v}(t) = \dfrac{d}{dt}\mathbf{r}(t)$
   - (2 pts) find the **unit** tangent vector $\mathbf{T}(t)$
   - (3 pts) find the principal **unit** normal vector $\mathbf{N}(t)$
   - (2 pts) find the binormal vector $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

4. (5 pts each) Determine if the series is absolutely convergent, conditionally convergent or divergent.

   (a) $\sum_{k=1}^{\infty}(\frac{k+1}{k})^k$

   (b) $\sum_{k=1}^{\infty}\frac{2}{1+e^k}$

   (c) $\sum_{k=1}^{\infty}(\sqrt[k]{2}-1)$

   (d) $\sum_{k=1}^{\infty}\frac{\cos k\pi}{k+1}$

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5. (5 pts) Determine the radius of convergence of the power series.

$$\sum_{k=1}^{\infty} \frac{(3k)!}{(k!)^3} x^k.$$

6. (5 pts) For $f(x) = e^x$, find the Taylor polynomial of degree 3 expanded about $x = 0$.

7. (15 pts) Given that $\dfrac{1}{1+x} = \displaystyle\sum_{k=0}^{\infty} (-1)^k x^k, \quad \text{for } -1 < x < 1,$

   - (6 pts) find the power series representation of $\frac{1}{1+x^2}$ and determine the radius and interval of convergence.

   - (6 pts) Find the power series representation of $\tan^{-1}(x)$ and determine the radius and interval of convergence.

   - (3 pts) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{1}{2k+1} = ?$

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8. (5 pts) Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{6x^3y}{x^6+y^2}$$

9. (5 pts)

$$\lim_{(x,y)\to(2,3)} \frac{6xy}{x^2+y^2} =?$$

10. (10 pts) Find the indicated partial derivatives.

$$f(x,y) = x^y - 3xy, \quad x, y > 0; \quad f_x, \quad f_y, \quad f_{xy}, \quad f_{xx}$$

11. (5 pts) Find the equation of the tangent plane to the surface at the given point.

$$z = x^2 - y^2 + 1 \quad \text{at } (2, 1, 2)$$