Quiz 1
Feb. 23, 2006

1. (5pts) Determine if the sequence converges or diverges. If it converges, determine the limit.

\[ a_n = \frac{2n^2 + 1}{n^2 - 3} \]

Ans:

\[ \lim_{n \to \infty} \frac{2n^2 + 1}{n^2 - 3} = \lim_{n \to \infty} \frac{2 + 1/n^2}{1 - 3/n^2} = 2 \]

The sequence converges to 2.

2. (5pts) Determine if the series converges or diverges. If it converges, find the sum.

\[ \sum_{n=1}^{\infty} \left( \frac{1}{3^n} + \frac{2}{k} \right) \]

Ans:

(i) \( \sum_{n=1}^{\infty} \frac{1}{3^n} \) converges (geometric series)

(ii) \( \sum_{n=1}^{\infty} \frac{2}{k} = 2 \sum_{n=1}^{\infty} \frac{1}{k} \) diverges (harmonic series)

Therefore \( \sum_{n=1}^{\infty} \left( \frac{1}{3^n} + \frac{2}{k} \right) \) diverges.

3. (0pt) Achilles and the tortoise—"You can never catch up."

"In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead."

(Aristotle Physics VI:9, 239b15)

Being very fast, Greek hero Achilles gives the Tortoise a head start. But that would be a big mistake, claims Zeno, for then Achilles could never catch up. Why not? The proof is that by the time Achilles reaches the Tortoises starting point \( S_1 \), the Tortoise will have moved a little to \( S_2 \), but then by the time Achilles reaches \( S_2 \) the Tortoise will have moved on to \( S_3 \), but then by the time ...

Write your solutions as complete as possible. Working time: 10 minutes.