Calculus II Name:______________________________
Student ID:____________________________

Quiz 2

Mar. 2, 2006

1. (5pts) Determine convergence or divergence of the series.

$$\sum_{k=1}^{\infty} \frac{k+1}{k^3+k+1}$$

Ans:
(1.) Limit Comparison Test:
(i) $\lim_{k\to\infty}(\frac{1}{k^2})/\frac{k+1}{k^3+k+1} = \lim_{k\to\infty}\frac{k^3+k+1}{k^3+k} = \lim_{k\to\infty}\frac{1+1/k^2+1/k^3}{1+1/k^2} = 1 \neq 0$
(ii) $\sum_{k=1}^{\infty}\frac{1}{k^2}$ converges (p-series with $p = 2 > 1$)
By Limit Comparison Test, $\sum_{k=1}^{\infty}\frac{k+1}{k^3+k+1}$ converges.

(2.) Comparison Test:
(i) Set $a_k = \frac{k+1}{k^3+k+1}$ and $b_k = \frac{1}{(k-1)^2}$, for $k > 1$.
(ii) For $k > 1$, $\frac{k+1}{k^3+k+1} - \frac{1}{(k-1)^2} = \frac{k^3-k^2-k+1-(k^3+k+1)}{(k^3+k+1)(k-1)^2} = \frac{-k^2-2k}{(k^3+k+1)(k-1)^2} < 0 \Rightarrow 0 < a_k < b_k$
(iii) $\sum_{k=2}^{\infty}\frac{1}{(k-1)^2}$ converges (p-series with $p = 2 > 1$)
By Comparison Test, $\sum_{k=2}^{\infty}\frac{k+1}{k^3+k+1}$ converges and $\sum_{k=1}^{\infty}\frac{k+1}{k^3+k+1}$ does, too.
Note: $\frac{k+1}{k^3+k+1} > \frac{1}{k^2}$ for $k > 1$.

(3.) Integral Test is not recommended

2. (5pts) Determine convergence or divergence of the series.

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{3}{k+2}$$

Ans:
(i) Set $a_k = \frac{3}{k+2}$. $a_k = \frac{3}{k+2} > 0$ for $k \geq 1$
(ii) $\lim_{k\to\infty} a_k = \lim_{k\to\infty}\frac{3}{k+2} = 0$
(iii) $k+1 > k \Rightarrow (k+1)+2 > k+2 \Rightarrow \frac{1}{(k+1)+2} < \frac{1}{k+2} \Rightarrow a_{k+1} = \frac{3}{(k+1)+2} < \frac{3}{k+2} = a_k$
$\Rightarrow \{a_k\}_{k=1}^{\infty}$ is decreasing.
By Alternating Series Test, $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{3}{k+2}$ converges.

Write your solutions as complete as possible. Working time: 15 minutes.