Calculus II Name:____________________________

Student ID:________________________

Quiz 4

Mar. 16, 2006

1. (5pts) Determine the radius and interval of convergence of the power series.

$$\sum_{k=1}^{\infty} \frac{k}{2^k}(x-2)^k$$

Ans:

$$\begin{aligned}\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| &= \lim_{k\to\infty} \frac{(k+1)|x-2|^{k+1}}{2^{k+1}} \frac{2^k}{k|x-2|^k} \\ &= \lim_{k\to\infty} \frac{k+1}{k} \frac{|x-2|}{2} \\ &= \frac{|x-2|}{2}\end{aligned}$$

By Ratio Test, the series converges for $\frac{|x-2|}{2} < 1$ or $|x-2| < 2$ and diverges for $\frac{|x-2|}{2} > 1$ or $|x-2| > 2$ so the radius of convergence is 2.
At $x = 4$, the series is

$$\sum_{k=1}^{\infty} \frac{k}{2^k}(2)^k = \sum_{k=1}^{\infty} k.$$

Since $\lim_{k\to\infty} k \neq 0$, by Divergence Test (kth-term test), the series diverges.
At $x = 0$, the series is

$$\sum_{k=1}^{\infty} \frac{k}{2^k}(-2)^k = \sum_{k=1}^{\infty} (-1)^k k.$$

Since $\lim_{k\to\infty} (-1)^k \neq 0$, by Divergence Test (kth-term test), the series diverges.
Therefore, the interval of convergence is $(0, 4)$.

2. (5pts) For $f(x) = \sin x$, find the Taylor polynomial of degree 2 expanded about $x = \frac{\pi}{2}$.

Ans:

$$\begin{aligned} f(x) &= \sin x & f(\pi/2) &= 1 \\ f'(x) &= \cos x & f'(\pi/2) &= 0 \\ f''(x) &= -\sin x & f''(\pi/2) &= -1 \end{aligned}$$

$$P_2(x) = f(\pi/2) + \frac{f'(\pi/2)}{1!}(x - \frac{\pi}{2})^1 + \frac{f''(\pi/2)}{2!}(x - \frac{\pi}{2})^2 = 1 - \frac{1}{2}(x - \frac{\pi}{2})^2$$

Write your solutions as complete as possible. Working time: 15 minutes.