Calculus II Midterm 1 Thursday, March 30, 2006

No calculator is allowed. No credit will be given for an answer without reasoning

1. (8 pts) Determine if the sequence converges or diverges. If it converges, determine the limit.
$$a_n = \frac{n^3+1}{2n^3-3n+2}$$

2. (8 pts) Determine the convergence of the series by the Integral Test.
$$\sum_{k=1}^{\infty} \frac{1}{k^2}$$

3. (8 pts each) Determine if the series is absolutely convergent, conditionally convergent or divergent.
   - (a)
   $$\sum_{k=1}^{\infty} \left(\frac{1}{2^k} - \frac{2}{k}\right)$$
   - (b)
   $$\sum_{k=1}^{\infty} \frac{k+9}{k^3+k^2+1}$$
   - (c)
   $$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sqrt{k}}{3k+2}$$
   - (d)
   $$\sum_{k=1}^{\infty} (-1)^k \frac{k^3}{(2k)!}$$

4. (8 pts) Determine the radius and interval of convergence of the power series.
$$\sum_{k=1}^{\infty} \frac{k}{2^k}(x-1)^k$$

5. (8 pts) For $f(x) = \cos x$, find the Taylor polynomial of degree 3 expanded about $x = \frac{\pi}{2}$.

6.
   - (8 pts) Given that
   $$\frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k, \quad \text{for } -1 < x < 1,$$
   find the Taylor series of $\frac{1}{1+x^2}$ and $\tan^{-1}(x)$. Determine the corresponding radius and interval of convergence.
   - (2 pts)
   $$\sum_{k=0}^{\infty} (-1)^k \frac{1}{2k+1} = ?$$

7. (8 pts) Use Euler-Fourier formulas to find the Fourier series of the function on the given interval.
$$f(x) = -x, \quad [-\pi, \pi]$$

8. (8 pts) For the equation below, find the recurrence relation and general power series solution of the form $y = \sum_{n=0}^{\infty} a_n x^n$.
$$y'' + 2xy' + 2y = 0$$

9. (8 pts) For $\mathbf{a} =< 2, -1, 3 >$ $\mathbf{b} =< 0, 2, 4 >$, find $\mathbf{p}roj_b a$.

10. (8 pts) Find the parametric equation of the line through $(1, 2, 3)$ and parallel to $< 2, 1, 4 >$.

- Double-Angle

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

- Derivative formulas

$$\begin{aligned}
\tfrac{d}{dx}\sin^{-1}x &= \frac{1}{\sqrt{1-x^2}},\\
\tfrac{d}{dx}\cos^{-1}x &= -\frac{1}{\sqrt{1-x^2}},\\
\tfrac{d}{dx}\tan^{-1}x &= \frac{1}{1+x^2},\\
\tfrac{d}{dx}\cot^{-1}x &= -\frac{1}{1-x^2},\\
\tfrac{d}{dx}\sec^{-1}x &= \frac{1}{|x|\sqrt{x^2-1}},\\
\tfrac{d}{dx}\csc^{-1}x &= -\frac{1}{|x|\sqrt{x^2-1}}
\end{aligned}$$

- Euler-Fourier formulas:

$$\begin{aligned}
f(x) &= \frac{a_0}{2} + \sum_{k=1}^{\infty}[a_k\cos(kx) + b_k\sin(kx)],\\
a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx,\\
a_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx, \quad \text{for } k = 1, 2, 3, \ldots\\
b_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx, \quad \text{for } k = 1, 2, 3, \ldots
\end{aligned}$$