Calculus II Midterm 2 Thursday, May 15, 2006

No calculator is allowed. No credit will be given for an answer without reasoning

1. Given that
$$\mathbf{r}(t) =< e^{2t}, t^2 - t, \cos 2t >,$$
calculate
   - (2 pts)
   $$\lim_{t\to 0} \mathbf{r}(t) = ?$$
   - (2 pts)
   $$\frac{d}{dt}\mathbf{r}(t) = ?$$
   - (4 pts)
   $$\int_1^s \mathbf{r}(t)\, dt = ?$$

2. Given the position function
$$\mathbf{r}(t) =< \sin 2t, \cos 2t, t >,$$
   - (2 pts) find the velocity,$\mathbf{v}(t)$
   - (2 pts) find the acceleration,$\mathbf{a}(t)$
   - (6 pts) find the unit tangent vector $\mathbf{T}(t)$ and the principal unit normal vector $\mathbf{N}(t)$
   - (3 pts) find the binormal vector $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$
   - (3 pts) find the curvature $\kappa = \frac{||\mathbf{T}'(t)||}{||\mathbf{r}'(t)||}$
   - (4 pts) find the tangential and normal component of acceleration $\mathbf{a}(t)$, *i.e.* find a_T and a_N of
   $$\mathbf{a}(t) = a_T\mathbf{T}(t) + a_N\mathbf{N}(t)$$

3. (8 pts) Show that the limit does not exist.
$$\lim_{(x,y)\to(0,0)} \frac{6x^3y}{x^6 + y^2}$$

4. (10 pts) Show that the limit exists. (You can use polar coordinates)
$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2 + y^2}$$

5. (8 pts) Find the indicated partial derivatives.
$$f(x, y) = x^3 - 3xy + y^3; \quad f_x, \quad f_y, \quad f_{xy}, \quad f_{xx}$$

6. (8 pts) Find the equation of the tangent plane to the surface at the given point.
$$z = x^2 - y^2 + 1 \quad \text{at } (2, 1, 2)$$

7. (8 pts) For a differentiable function $g(u, v) = f(x(u, v), y(u, v))$ with $x(u, v) = u\cos v$ and $y(u, v) = u \sin v$ and where f_{xy} and f_{yx} are continuous, compute f_u, f_v and f_{uu}.
$$f(x, y) = 4x^2y^3$$

8. (8 pts) Given that $f(x, y) = x^2 - y^2$, find the gradient of $f(x, y)$ at $(2, 1)$ and compute the directional derivative of f in the direction of $\mathbf{u} =< \frac{1}{2}, \frac{\sqrt{3}}{2} >$

9. (10 pts) For $f(x, y) = x^3 - 3x + \frac{1}{3}xy^2$, locate all critical points and classify them.

10. (12 pts) Find the maximum and minimum of the function $f(x, y)$ subject to the constraint $g(x, y) \leq c$.
$$f(x, y) = x^3 - 3x + \frac{1}{3}xy^2, \quad \text{subject to } x^2 + y^2 \leq 3$$